Three-phase Load Signature: a wavelet-based approach to power analysis

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This paper presents an analysis of the load signature problem based on Park-domain instantaneous power definition. In particular, the Park power vector is analyzed in the wavelet domain. The analysis is performed by using two different bases to obtain information on the selection criteria. Different loads are analyzed and different challenges are discussed. The transient data are obtained via simulation by using a particular simulation platform called Virtual Test Bed.

1. Introduction

In recent days the load signature analysis has assumed a growing role. In effect, it is rather easy to identify applications where the information about the working conditions of the load are critical for the safety of the system itself. An interesting case is provided by ship power systems where the limited availability of power makes the load monitoring extremely important. However, this is not the only case: there is an increasing interest for example also in the field of building automation. Last but not least the recent trend towards a more distributed power generation makes the network condition completely different from the traditional concentrated power generation and the gathering of information about the load more important.

However, the scientific interest for the load signature is not new. One of the common elements of previous efforts in this field is the need to limit the impact of the monitoring system on the rest of the plant. In this respect it is worth mentioning the idea introduced in [1] where the problem is tackled thanks to a device called Non Intrusive Load Monitor (NILM). This acronym perfectly summarizes the aforementioned need and can be considered the starting point for the design of a general-purpose monitoring system for load signature analysis.

However, the first reference to a non-intrusive approach to monitoring dates back to the early 80s’ for application in residential building by Schewepe and Hart from MIT. Summarizing, a non-intrusive approach is capable of drawing conclusions with limited available information, in particular determining the critical quantities simply through terminal measurements with limited insight information.

This condition, as also described in [2], is becoming more and more important while the commercial applications of systems such as Variable Speed Electrical Drives become more and more integrated and less accessible. In effect, a sensor-less system such as the one described in [3] is another perfect example of non-intrusive approach to the problem.

The exploitation of limited information concerning the load requires an increasing complexity on the data processing of the available data.

To cope with the described requirements, in the following, we will introduce a short review of the possible approaches for the measurement stage and then we will focus on the application of the concept of the instantaneous power in the Park domain.

In particular a time-frequency analysis of the quantity is considered as significant tool to gather information. The theoretical assumption will be documented through a wide set of simulation scenarios that will provide the reference data for the post-processing.

2. The measurement approach

2.1 Available information

Keeping in mind the hypothesis of non-intrusive approach it is easy to understand that list of available data is restricted to:

- Currents
- Voltages
- Instantaneous power

The first and the third represent a classical solution. In particular many works show how to gather information from the transient current [4,5].

The hypothesis of simply using the voltage is still under consideration. The main attraction is the simplification in the measurement stage; the main drawback is the higher level of difficulty on extracting significant information in the post-processing [6].

In this paper we focus on a solution based on instantaneous power in the Park domain. The vector characteristic of this quantity opens the way to interesting analysis with respect to the simple time domain instantaneous total power.

2.2 Power definition

For sake of completeness let us quickly recall the main definitions concerning the instantaneous power in the Park domain.

The Park transformation can be applied generally speaking to any multi-phase system. In the simplest case we can focus on a three-phase scenario. Given the three phase quantities \( x_a, x_b, \) and \( x_c \) the correspondent Park domain quantity is obtained applying a suitable matrix \( T \) to the vector \( X \) defined as:
The vector \( \mathbf{X} = [x_a, x_b, x_c] \) so that we obtain a new quantity in the new domain as: 
\[ \mathbf{X}_p = T \mathbf{X} \]

The matrix \( T \) can be defined according to different criteria including the relative speed between the three-phase phasor and the reference Park system \([7]\). In the following we adopt the following definition:

This matrix, that holds for the case of fixed Park reference, defines an orthonormal transformation so that:

\[ T^{-1} = T^t \]

Given the definition of the transformation we introduce the power in the Park domain as the product between the voltage Park vector and the conjugate of the current Park vector:

\[ \mathbf{p} = \mathbf{v}^* = (v_d + jv_q)(i_d - ji_q) =
\]

Two comments complete this brief introduction:

- In the case of a steady-state sinusoidal system the real and imaginary part of the Park power are coincident with the classical active and reactive power
- Because of the properties of the transformation the power does not depend on the specific reference adopted (this is a classical propriety of any tensor analysis)

### 2.3 A simple example

A simple example can help understanding the properties of the Park power vector providing interesting information also for what it concerns the post-processing oriented towards the load signature analysis. Let us focus on a very simple case: a three-phase system composed by an ideal source and a linear time-invariant resistive-inductive load. This basic case can be easily analyzed in closed-form thus opening the way to interesting considerations.

The main equation in the Park domain can be written as:

\[ \mathbf{v} = \mathbf{R} \mathbf{i} + \mathbf{L} \frac{d \mathbf{i}}{dt} \]

Let us now suppose the source is ideal, balanced and perfectly sinusoidal. The Park voltage vector will be:

\[ \mathbf{v} = \mathbf{V}_e e^{j\omega t} \]

where the amplitude of the vector \( V \) is the rms value of the line to line voltage and \( \omega \) is the radiant-frequency of the system.

The following solution holds:

\[ \mathbf{i} = \mathbf{K} e^{-Rt/L} + \mathbf{I}_0 e^{j\omega t} \]

where:

- \( \mathbf{K} \) is a vector that depends on the initial conditions
- \( \mathbf{I}_0 \) is the Park vector describing the steady state solution.

Calculating the power:

\[ \mathbf{p} = \mathbf{v}^* T \mathbf{K} e^{-Rt/L} e^{j\omega t} \]

Summarizing we can state the following:

- Whenever we have a transient in a three-phase system the Park power will react with an equivalent transient
- While current and voltage still have a sinusoidal steady-state, the power has a constant steady state
- While current and voltage have an exponential decay transient, the power has an oscillatory transient with amplitude decaying exponentially

### 3. The load signature challenge

After introducing the quantities under analysis we can summarize the list of challenges that we expect to face in a real application:

- Systems can present transient characteristics
- Transient duration can widely vary among different loads
- Some systems are basically always working under transient conditions
- Non linear loads introduce more terms in the power flow

This set of problems defines the main challenges we are going to face. Basically we can state the following:

- The low harmonic content of the Park power vector contains the main information about power flow
- Time evolution of the low harmonic content is usually an interesting element to identify load characteristics
- Higher harmonic content can be extremely significant mostly for non linear loads

This list of considerations calls for the following conclusions:

- A simple frequency domain analysis is not enough for load signature analysis
- A wavelet approach seems to be a reasonable solution considering the higher degree of freedom given by the multi-resolution property
- The selection of the bases has to performed mostly in consideration of the high capability of identifying the low frequency content
In the following sections we are going to investigate the results obtained by using two different bases:

- Haar wavelet
- Malvar wavelet

This selection has been performed for the following reasons [8]:

- Haar is the simplest base and the analysis is really computationally simple
- Both Haar and Malvar can easily detect low frequency content
- Malvar is defined by a mother wavelet that can be defined as a quasi-cosinusoidal waveform and then it is closer to the classical harmonic analysis

Together with the selection of the base comparing the two proposed candidates we will also investigate the role of the sampling time and the size of the buffer adopted for the transformation. The results are obtained running Matlab routines in parallel with Virtual Test Bed simulations [9]

The different load topologies considered for the analysis are:

- Three phase linear RL load
- Three phase Graetz Thyristor bridge with RL load
- Induction motor starting
- Cycloconverter drive

4. Wavelet basis

Before the description of the example we want to briefly summarize the main mathematical characteristics of the two selected bases. This introduction is supposed to point out the element of distinctions allowing a better understanding of the simulation results.

The time-frequency analysis is necessary when transient signals or signals with singularities are involved. Unfortunately the resolution in time and frequency cannot be independently arbitrarily good. Provided that the best we can do has to comply with the Heisenberg principle, the two resolutions can be set. The choice of time and frequency resolution turns out to be a trade-off. A better time resolution results in a poorer frequency resolution and vice-versa. Referring to the windowed Fourier transform, for example, it’s intuitive to see that a window with a large support and consequently a more concentrated frequency content, allows a better frequency resolution but results in a poor time resolution. Once the window is chosen, the time-frequency resolution is fixed and is identical at every frequency. The wavelet approach allows more flexibility. Let’s refer to the discrete wavelet transform. A wavelet base can be built in a way that the resolution is different at different frequencies. For example the frequency resolution can be better at low frequency and time resolution better at high frequency. This structure is convenient if we want to identify with good precision the fundamental frequency of an electrical signal and we want to identify with good time precision the presence of sudden glitches. The second peculiarity is related to the multi-resolution analysis. The analysis performed with wavelets allows the representation of a signal as the superposition of a coarser representation and subsequent finer details. The coarseness of the representation and the level of detail can be set at convenience.

From the computational point of view, the joining of the harmonic analysis perspective and the filter bank theory results in a very efficient way to compute the wavelet transform through filtering the signals with finite length discrete filters. The computation of the wavelet coefficients of a signal are reduced to the computation of averages and differences of the coefficients previously computed. These operations can be efficiently implemented in a cascade of filter banks whose inputs are the samples of the signals. The sampling sets the limit to the finesse of the details. The Haar wavelet basis is an orthogonal basis used in the applications described in the following sections. The Haar wavelet is defined as:

$$\psi(t) = \begin{cases} 
1 & 0 \leq t < 1/2 \\
-1 & 1/2 \leq t < 1 \\
0 & \text{otherwise} 
\end{cases}$$

The other functions of the basis are obtained with rescaling and shifting of the so-called mother wavelet previously defined. In particular the other functions of the basis are obtained according to:

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k)$$

so that the scaling factor $j$ turns the length of the support to $2^j$, $k$ is the shift factor and $2^j$ is the normalization factor.

The scaling function of the Haar wavelet system is defined as:

$$\phi(t) = \begin{cases} 
1 & 0 \leq t < 1 \\
0 & \text{otherwise} 
\end{cases}$$

This orthogonal system allows the decomposition of any $L_2(\mathbb{R})$ function into the sum of a coarse representation related to the scaling function, with an averaging meaning, and a sum of finer and finer details related to the wavelets. This is the core of multi-resolution analysis. A convenient aspect of the decomposition calculation is that, once the coarsest representation of the function is computed, the rest of the decomposition only involves sums and differences in discrete calculations.

Malvar’s wavelets are part of the so-called Local Cosine Bases [10]. They represent an orthogonal base that can be summarized by the following expression:

$$\phi_{j,k}(t) = \sqrt{\frac{2}{L_j}} w_j(t) \cos \left[ \frac{\pi}{L_j} \left( k + \frac{1}{2} \right) (t - a_j) \right]$$

where $k=0,1,2 \ldots$ and $j \in \mathbb{Z}$, $a_j$ is an increasing sequence of real numbers and the window function $w_j(t)$ is centered around the interval $[a_j, a_{j+1}]$. Finally:

$$L_j = a_{j+1} - a_j$$
Different wavelet families can be obtained defining different window functions. Through the introduction of a suitable smooth window a set of smooth local time-domain function is obtained. Their exponential decay can be arbitrary depending on the smoothness and differentiability of the window itself. As a result of that a local modulated basis with good time-frequency localization is defined. From this perspective, we can say that we have a base with many elements of similarities with a windowed Fourier transform, but with better characteristics of localization both in time and frequency. This propriety of smoothness will be widely exploited in the rest of the paper to obtain compact spectra of the power function even under widely distorted conditions.

5. The RL linear load

Let us now focus on the first example. For this case we have the possibility to compare the simulation results with the actual theoretical prediction. Figure 1 shows the VTB schematic for this scenario. A three-phase source is connected to a three-phase load. In the middle a power sensor unit is inserted. This block has been specifically developed for this application and presents as outputs the two components of the Park power vector. Those two signals are then sent to Matlab for run-time processing, by using the VTB-Matlab interface [11]. Running the simulation a Matlab script will produce the 3D maps of the wavelet transformation both for the real and the imaginary component. The following degree of freedom will be analyzed:

- Role of the sampling time
- Role of the buffer size

Looking at Figure 3 and Figure 4 it is interesting to realize that the wavelet analysis is able to spot from the very beginning the steady state conditions even if the transient just started. Those two figures in fact report the results of the very first 256 points of simulation. Furthermore, as theory states, when steady state conditions are reached the spectra are completely flat a part from the coefficient related to the father wavelet.
the other components fell down to zero. The imaginary component gives the same trend. As result of that we can state the following:

- The Haar base is able to detect from the very beginning the final steady state condition. This will appear as the coefficient driving the father wavelet. This is really important for very slow transient to have an immediate figure of merit of the requested level of power.
- Any transient effect appear on the other wavelet order, so that the total energy spectra can be considered a significant index of the distance from steady state conditions.

Figure 5: Steady state condition of the real part (x-axis samples, y-axis wavelet order)

The same experiment can be repeated with the Malvar base. In this case we adopt a sampling time of 1 ms over 16 intervals of 10 points each. The period of analysis is then longer than in the Haar case. Looking at the results (see Figure 6 and Figure 7) we can conclude that with this set of parameters the Malvar base is not able to extract with the same accuracy the steady state value, but on the other side it contains a more compact spectra. It is interesting to underline that only the first three orders of wavelets contributes to the spectra. This results is an interesting element for the analysis of more complex loads where distortion may appear.

A more compact spectrum distribution will facilitate any algorithm for load identification under noisy conditions. On the other hand, it is also interesting to observe the results obtained by using a 2 ms sampling time. In this case for each window of analysis we have a full period at 50 Hz. Looking at the wavelet spectrum of the real component in this case (see Figure 8) we can easily identify the transient component superimposed to the final average contribution. Once more the spectrum is still very compact.

Figure 6: real part of Park power analyzed with Malvar wavelets (x-axis intervals, y-axis wavelet order)

Figure 7: imaginary part of Park power analyzed with Malvar wavelets (x-axis intervals, y-axis wavelet order)

Figure 8: real power component with a sampling time of 2 ms (x-axis intervals, y-axis wavelet order)

6. A reversible Graetz bridge

However, other interesting conclusions can be obtained for loads introducing a significant distortion. In this case it is not easy to obtain a priori information but the simulation analysis allows the determination of interesting characteristics of the behaviour of the selected wavelet base.

Let us now focus on a Graetz bridge with a resistive inductive load. In this case the current on the input of the converter is highly distorted with a quasi-square wave behaviour. The analysis of the power spectra is now more complicated and the extraction of significant signatures more challenging.
Figure 9 reports the VTB schematic adopted for this group of testing. The firing angle of the converter can be controlled through the control input and it will be considered constant during a single data acquisition.

![VTB schematic](image1)

**Figure 9: VTB schematic for the bridge case**

As results in Figure 10 and Figure 11 show, the most of the properties demonstrated in the linear case are preserved. The main power contribution is immediately identified even though the higher order of wavelets are now really complicated to manage in a compact form. Repeating the same experiment with the Malvar wavelets we obtain a results that is still perfectly coherent with the results of the first linear case. Looking at Figure 12, the Malvar base perfectly isolate the main contribution of the transient that is a simple DC first order transient. Furthermore the other components are reasonably constant in time giving a significant information for the load identification.

![Power with Haar base and Graetz bridge load](image2)

**Figure 10: Park real power with Haar base and Graetz bridge load (x-axis samples, y-axis wavelet order)**

![Power with Malvar base and Graetz bridge load](image3)

**Figure 12: Park real power with Malvar base and Graetz bridge load (x-axis intervals, y-axis wavelet order)**

### 7. An induction machine case

An induction machine startup transient is another interesting load case to consider. The main challenge in this case is the presence of transients with quite different time constant ranging from the stator flux evolution to the mechanical system dynamic. Let us focus on the Malvar case. From this perspective it is interesting to compare the results in Figure 14 with those reported in Figure 15.

The first snapshot is obtained at the very beginning of the startup. First of all, even for this case, the spectrum is really compact. Furthermore the low frequency components clearly identify the fast torque ripple typical of the very beginning of the transient while the rotor flux is still not well established.

The second snapshot, viceversa, shows a clear slow transient in the power absorption that is typical of the final part of the start-up process. The evolution is now clearly driven by the mechanical time constant.

![Schematic for the induction machine case](image4)

**Figure 13: The schematic for the induction machine case**
8. A cycloconverter case

This last case represents the more challenging for the following reasons:

- The output frequency changes with time making synchronization impossible to achieve
- The input current is highly distorted by the Graetz bridge operation.

The schematic adopted for this test is reported in Figure 16. The drive is supposed to operate in open-loop with a simple V/Hz modulation strategy to determine the steady-state speed [12].

In this case, it is interesting to have a look at the time evolution of the real power before going into the details of the wavelet analysis.

As theoretically anticipated the signal is extremely noisy. Nevertheless the Malvar base is able to extract significant information about the load.

For example, it is interesting to underline the quasi-sinusoidal evolution of the main wavelets components versus time. This could be easily justified interpreting the current as the result of an amplitude modulation at low frequency with respect to the 50 Hz component. This characteristic is clearly a possible way to extract information about the drive operation.

Furthermore, looking at the results in Figure 20, it is still clear that even under extremely noisy conditions, the wavelet transform identify the main component of the power and then the operating condition of the drive.
The authors are now planning to design algorithms able to automatically extract the significant information from the wavelets.

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BIBLIOGRAPHY