Physics-Based Model of IGBT Including MOS Side Two-Dimensional Effects

L. Lu, A. Bryant*, E. Santi, J.L. Hudgins**, P.R. Palmer***

Department of Electrical Engineering, University of South Carolina, Columbia, SC 29208, Email: santi@engr.sc.edu
* School of Engineering, University of Warwick, Coventry CV4 7AL, United Kingdom, Email: a.t.bryant@warwick.ac.uk
** Department of Electrical Engineering, University of Nebraska, Lincoln, NE 68588-0511
*** Engineering Department, Cambridge University, Trumpington Street, Cambridge CB2 1PZ, UK

Abstract—An existing physics-based model, which has proven accurate for inductive turn-off and inductive turn-on simulations, is modified to account for lateral-gate IGBT two-dimensional effects at the MOS end of the drift region. The modification is based on a steady-state solution of carrier distribution in the JFET region of the IGBT. The accuracy of this solution is verified through a set of finite-element simulations. The improved accuracy of the modified model in terms of on-state forward drop and voltage tail at turn-on is verified through comparison with experimental results.

Keywords—Power semiconductor modeling, IGBT model, physics-based model

I. NOMENCLATURE

- \( A \)  Device area (cm²)
- \( a_i \)  Ratio of IGBT inter-cell to total die area
- \( D \)  Ambipolar diffusivity (cm²s⁻¹)
- \( D_n \)  Electron diffusivity (cm²s⁻¹)
- \( D_p \)  Hole diffusivity (cm²s⁻¹)
- \( h_p \)  P+ emitter recombination coefficient (cm⁴s⁻¹)
- \( I_{MOS} \)  IGBT MOSFET drain current (A)
- \( I_{n1} \)  Electron current at anode end of carrier storage region (A)
- \( I_{n2} \)  Electron current at cathode end of carrier storage region (A)
- \( I_{p1} \)  Hole current at anode end of carrier storage region (A)
- \( I_{p2} \)  Hole current at cathode end of carrier storage region (A)
- \( J_{p2} \)  Hole current density at cathode end of carrier storage region (Acm⁻²)
- \( J \)  Maximum current density (Acm⁻²)
- \( K \)  Gain used to determine voltage \( V_{MOS} \) as a function of carrier concentration \( p \)
- \( K_p \)  IGBT MOSFET transconductance (AV⁻²)
- \( L \)  IGBT cell half-width (µm)
- \( I_m \)  IGBT inter-cell half-width (µm)
- \( N_B \)  Drift region (base) doping concentration (cm⁻³)
- \( p \)  Ambipolar carrier density (cm⁻³)
- \( P_g \)  Ambipolar carrier density along edge of accumulation layer (cm⁻³)
- \( p_{a1} \)  Ambipolar carrier density at anode end of carrier storage region (cm⁻³)
- \( p_{a2} \)  Ambipolar carrier density at cathode end of carrier storage region (cm⁻³)
- \( p_{a2p} \)  Ambipolar carrier density at cathode end of carrier storage region under p-well (cm⁻³)
- \( p_{a2T} \)  Ambipolar carrier density at cathode end of carrier storage region at gate overlap (cm⁻³)
- \( q \)  Unit electron charge (=1.6x10⁻¹⁹C)
- \( \tau \)  High-level lifetime (s)
- \( V_{CE} \)  IGBT collector-emitter voltage (V)
- \( V_{GE} \)  IGBT MOSFET gate-source voltage (V)
- \( V_{MOS} \)  IGBT MOSFET drain-source voltage (V)
- \( V_{th} \)  IGBT MOSFET threshold voltage (V)
- \( W_B \)  Drift region (base) width (µm)
- \( W_P \)  Depth of P-well (µm)
- \( x_1 \)  Boundary position of anode end of carrier storage region (µm)
- \( x_2 \)  Boundary position of cathode end of carrier storage region (µm)

II. INTRODUCTION

Accurate IGBT models are needed in order to accurately simulate switching waveforms, estimate device stresses, and predict switching and conduction losses in converter applications. A complete physics-based electro-thermal IGBT circuit simulator model has been presented before [1-2]. Its
high accuracy has been validated through comparison with experimental results for inductive turn-off and inductive turn-on transients [3]. Its usefulness is enhanced by its practical parameterization procedure and reasonable simulation speed [4].

The model utilizes a one-dimensional Fourier series solution of the ambipolar diffusion equation (ADE) in the drift region. The one-dimensional assumption is valid for most of the drift region, but it breaks down at the MOS end, where significant two-dimensional effects are present. The existing model fails to capture these effects and as a consequence gives inaccurate predictions statically [5] for the forward drop during conduction and dynamically for conductivity modulation at turn-on.

In this paper an improved model is proposed that takes into account the two-dimensional effects in a simple manner, preserving the simplicity of the one-dimensional ADE solution. This is done by providing an appropriate boundary condition for the ADE that includes the two-dimensional effects.

### III. PHYSICS-BASED IGBT CIRCUIT SIMULATOR MODEL

The behavior of conductivity modulated devices, such as pin diodes and IGBTs, depends heavily on the excess carrier (charge) distribution in the wide drift region. In modern devices, the charge profile has a 1D form over most of its volume. Thus, a 1D solution is adequate for the bulk of the device. Space-charge neutrality is maintained with the majority carrier profile closely matching the minority carrier profile (quasi-neutrality). Under these conditions, and assuming high-level injection, the charge dynamics are described by the one-dimensional ADE:

\[
D \frac{\partial^2 p(x,t)}{\partial x^2} = \frac{p(x,t)}{\tau} \frac{\partial p(x,t)}{\partial t}. \tag{1}
\]

A Fourier based solution for this equation has been proposed [1]. The 2nd order partial differential carrier diffusion equation is converted into an infinite set of 1st order linear differential equations with series coefficients \(p_0 \ldots p_k\) forming equivalent \(R_nC_n\) components. The representation requires the width of the undepleted region and the hole and electron currents at the boundaries of the region \((x_1 \text{ and } x_2)\), which provide the boundary conditions. The drift region model can be used for IGBTs provided appropriate boundary conditions are used. Physics-based electro-thermal models for IGBTs have been developed and are described in [2].

#### A. Validation of Fourier-based-solution Approach

The Fourier-based solution for the drift region carrier distribution has been validated by comparison with ATLAS finite element simulations. The evolutions of the carrier distribution during IGBT inductive turn off predicted by ATLAS and by the model are shown in Fig. 1 and Fig. 2. The results show good agreement. The biggest discrepancy is in the oscillations of the charge profile at time 3.0\(\mu\)s. This is due to the truncation of the Fourier series retaining only the first seven terms. Comparison of experimental and simulation results for IGBT inductive turn off is shown in Fig. 3 and for inductive turn-on in Fig. 4.

![Fig. 1 Charge profiles during inductive turn off ATLAS finite element simulation (Note: Collector metal at 0 um). The numbers indicate time in \(\mu\)s.](image1)

![Fig. 2 Charge profiles during inductive turn off circuit simulator model (Note: Collector metal at 0 um). The numbers indicate time in \(\mu\)s.](image2)

![Fig. 3 Experimental and simulation results for IGBT turn-off transient at 600V/100A under inductive load at 300K](image3)
where

The ADE boundary condition is:

\[ \text{boundary condition on the MOS side is established as follows.} \]

which does not take into account two-dimensional effects, is reviewed. Then the two-dimensional effects are briefly described and an improved boundary condition is introduced.

In the existing physics-based model the drift region boundary condition on the MOS side is established as follows. The ADE boundary condition is:

\[ g(t) = \frac{\partial p(x,t)}{\partial x} \bigg|_{x=2} = \frac{1}{2qA} \left[ \frac{I_{n2} - I_{p2}}{D_n} - \frac{I_{p2}}{D_p} \right], \tag{2} \]

where \( D_n \) and \( D_p \) are the electron and hole diffusivity and \( I_{n2} \) and \( I_{p2} \) are the electron and hole currents at the MOS end of the drift region. Since the total current is known (in the sense that it is calculated by the simulator), specifying \( I_{n2} \) is sufficient to establish the boundary condition (2). The current \( I_{n2} \) is equal to the MOSFET current:

\[ I_{n2} = I_{\text{MOS}}. \tag{3} \]

This is calculated using the standard MOSFET equations for operation in the linear region and in saturation. For operation in the linear region, which is typical during on-state, the MOS current is given by (4).

\[ I_{\text{MOS}} = K_p \left[ V_{\text{MOS}}(V_{GE} - V_{th}) - V_{\text{MOS}}^2 / 2 \right] \tag{4} \]

The MOS voltage is given by (5) where \( K \) is a large gain and \( p_{n2} \) is the carrier concentration at the MOS end of the drift region calculated as part of the Fourier series solution of the ADE.

\[ V_{\text{MOS}} = \begin{cases} 0 & \text{for } p_{n2} > 0 \\ -Kp_{n2} & \text{for } p_{n2} < 0 \end{cases} \tag{5} \]

Equations (2), (3), (4), (5) and the Fourier series calculation of the carrier profile create a negative feedback loop that forces \( p_{n2} \) to be zero.

\[ p_{n2} \rightarrow (3) \rightarrow V_{\text{MOS}} \rightarrow (4) \rightarrow I_{\text{MOS}} \rightarrow (3) \rightarrow I_{n2} \rightarrow (2) \rightarrow p_{n2} \]

Notice that in order for the MOSFET current to be non-zero, voltage \( V_{\text{MOS}} \) must be positive (4), and therefore the carrier concentration \( p_{n2} \) at the MOS end of the drift region must be negative. In practice, since the gain \( K \) in equation (5) is large, the carrier concentration \( p_{n2} \) is approximately zero during conduction. In conclusion, the feedback adjusts the MOS electron current so that the carrier distribution in the drift region falls to zero at the MOS end.

We now investigate the carrier distribution in the IGBT drift region using an Atlas finite element simulation. The IGBT structure is pictorially represented in Fig. 5. It is a 270\( \mu \)m NPT lateral gate IGBT. Fig. 5 also shows the carrier flow in the on-state. At the collector side (anode) holes are injected from the p-emitter into the drift region and electrons flow in the opposite direction from the drift region into the p-emitter. At the MOS end (cathode) electrons and holes take two different routes: electrons flow vertically from the MOS channel into the drift region, and holes flow laterally into the p-well under the inversion channel created by the gate. We can look at the carrier distribution during the on-state by looking at a vertical cutline under the p-well and a vertical cutline under the gate (cutlines A and B respectively in Fig. 5). The carrier distributions are shown in Fig. 6 for the entire drift region. Notice that the carrier concentration drops to zero at the p-well, because the drift region p-well junction is reverse biased. However, this is not true at the accumulation layer under the gate overlap. The two distributions are identical from 0\( \mu \)m (p-emitter side) all the way to 240\( \mu \)m, validating the one-dimensional assumption for the ADE. However, on the MOS side the two distributions are different. Fig. 7 shows the detail of the carrier distributions at the MOS end of the drift region from 240\( \mu \)m to 270\( \mu \)m.

As explained above, the existing model forces the carrier distribution to drop to zero at the MOS end (\( p_{n2} = 0 \)). This does not agree with the finite element simulation results of Fig. 7. This inaccuracy causes two problems:

1. In the on-state the forward drop predicted by the model is too high. The biggest contribution to voltage drop in the drift region comes from the region close to the MOS end, where the carrier concentration is lower and there is less conductivity modulation.
2. At turn-on, the carrier dynamics are different. At the beginning of the turn-on interval the drift region is not conductivity modulated and is highly resistive causing a significant voltage drop. As charge accumulates in the drift region, this voltage drop decreases. This causes a voltage “tail” at turn-on, which is particularly evident for resistive turn-on. The model predicts too long a voltage tail, because \( p_{x2} \) is pinned at zero and it takes a long time for the charge to begin accumulating at the MOS end of the drift region.

The proposed solution is to retain the simplicity of the one-dimensional Fourier series ADE solution, but provide a different boundary condition at the MOS end, allowing carrier concentration \( p_{x2} \) to be non-zero. From Fig. 7, it appears that an appropriate boundary condition may be given by the weighted average of carrier concentrations \( p_{x2P} \) (under the p-well) and \( p_{x2T} \) (under the gate overlap):

\[
\bar{p}_{x2} = a_i \bar{p}_{x2T} + (1 - a_i) \bar{p}_{x2P},
\]

where \( a_i \) is the intercell area ratio, i.e., the ratio of the gate overlap area over the active area.

This equation can be used to calculate \( p_{x2P} \) from the value of \( \bar{p}_{x2} \) obtained from the Fourier solution, provided that \( p_{x2T} \) is known. Therefore, the model calculation sequence, originally given by (6), is modified as follows.

\[
p_{x2P} \xrightarrow{(4)} V_{MOS} \xrightarrow{(3)} I_{MOS} \xrightarrow{(2)} I_{x2} \xrightarrow{(1)} g(t) \xrightarrow{(0)} \text{Fourier} \xrightarrow{(6)} \bar{p}_{x2} \xrightarrow{(1)} p_{x2P}
\]

V. CALCULATION OF CARRIER CONCENTRATION \( p_{x2T} \)

In order to be able to use (7) to calculate carrier concentration \( p_{x2P} \) as required by the model calculation sequence (8), it is necessary to calculate carrier concentration...
$p_{xT}$ under the gate overlap. So the remaining problem is how to calculate $p_{xT}$. The chosen approach follows the development in Johnson [6] and Sheng [7]. In both papers an approximate steady-state solution of the ADE is found for the intercell region between p-wells in Fig. 5 (the so-called JFET region). Fig. 8 shows this region in detail, showing geometric dimensions and the coordinate axes used. The steady-state ambipolar diffusion equation is

$$D \left[ \frac{\partial^2 p(x,y)}{\partial x^2} + \frac{\partial^2 p(x,y)}{\partial y^2} \right] = \frac{p(x,y)}{\tau}$$

(9)

In order to simplify the analysis, recombination is neglected and equation (9) simplifies to the two-dimensional Laplace equation

$$\frac{\partial^2 p(x,y)}{\partial x^2} + \frac{\partial^2 p(x,y)}{\partial y^2} = 0$$

(10)

This is a good approximation given the fact that the long lifetime in IGBTs gives a large diffusion length, typically larger than the drift region width.

The carrier concentration $p_{xT}$ can be calculated as the average value of $p(x,y)$ along the base of the JFET region, i.e., along the line $y = 0$ to $y = l_m$ at $x = W_P$. Performing the integral using (11) and (12), the result is

$$p_{xT} = \frac{L}{6qD_p} \frac{l_m}{W_P} \frac{I_{p2}}{A}$$

(13)

The development by Sheng is similar. There, the boundary conditions are based on the carrier concentration along the edges of the intercell region, with a solution of the form:

$$p(x,y) = \frac{2}{\pi} P_g \arctan \left( \frac{y}{x} \right)$$

(14)

where $P_g$ is the carrier concentration along the edge of the accumulation layer, i.e., the line from $y = 0$ to $y = l_m$ at $x = 0$.

The current $I_{p2}$ can be calculated along the boundary between the p-well and the JFET region, i.e., the line from $x = 0$ to $x = W_P$ at $y = 0$. Some care is necessary while performing this integral because the integrand tends to infinity at the origin. This is resolved by assuming that no hole current flows in the 10% of the p-well depth nearest the gate oxide, and only integrating between $0.1W_P < x < W_P$). The result is:

$$I_{p2} = \frac{4qAD_p \ln(10)}{\pi L} P_g$$

(15)

The ln(10) term results from the aforementioned 10% assumption. The carrier concentration $p_{xT}$ can be calculated from (14) in exactly the same way as (13). Defining the following geometric constant

$$G \left( \frac{l_m}{W_P} \right) = \arctan \left( \frac{l_m}{W_P} \right) - \frac{W_P}{2l_m} \ln \left[ 1 + \left( \frac{l_m}{W_P} \right)^2 \right]$$

(16)

we obtain,

$$p_{xT} = \frac{2P_g}{\pi} G \left( \frac{l_m}{W_P} \right)$$

(17)

Finally, by combining (15) and (17) we obtain the desired relationship for $p_{xT}$:

$$p_{xT} = \frac{L}{2\ln(10)qD_p} G \left( \frac{l_m}{W_P} \right) \frac{I_{p2}}{A}$$

(18)

Notice that equations (13) and (18) are suitable equations to calculate $p_{xT}$. It is interesting to notice that even if the two derivations are different, these two equations have striking similarities. The three factors on the right hand side of these equations are similar: the first term is identical except for the numerical coefficient in the denominator, the second one is a function of the aspect ratio $l_m/W_P$ only, and the third term
represents the hole current density at the MOS end of the drift region. One interesting conclusion is that carrier concentration \( \rho_{x2T} \) is a linear function of hole current density. Notice that at the p-emitter end of the drift region the boundary condition is defined in terms of the \( h_p \) parameter and therefore the relationship is

\[
\frac{J_{pl}}{A} = qh_p\rho_{x1}^2
\]  

(19)

In this case current density is proportional to the square of the carrier concentration.

VI. Finite Element Validation of Expressions (13) and (18) for \( \rho_{x2T} \)

According to equations (13) and (18), \( \rho_{x2T} \) is directly proportional to hole current density and a function of JFET geometry. In order to test the accuracy of these expressions, finite element simulations of a lateral-gate IGBT are performed. A linear (striped) cell structure is simulated in order to be compatible with the geometry assumed by Johnson and Sheng in [6] and [7]. In case of different cell geometry, a transformation can be performed to transform it into an equivalent linear cell structure as described in [7]. The IGBT parameters used in the finite element simulation are listed in Table I.

In order to test the dependence with respect to hole current density \( J_{p2} \), finite element simulations are performed for different current densities and the results are plotted in Fig. 9. The carrier density is extracted from the finite element simulation exactly in the same way as it has been calculated in the previous section, i.e., as the average carrier density along a cutline from \( y = 0 \) to \( y = l_m \) at \( x = W_F \). This confirms that there is indeed a linear dependence.

The conclusion is that both Johnson and Sheng formulas capture the dependence on hole current density and geometry and appear to provide adequate accuracy for use in the IGBT model. The Sheng formula appears to be somewhat more accurate for the simulated IGBT, but the Johnson formula is actually implemented in the model due to its simplicity. The Sheng formula could be as easily implemented if desired.

VII. Validation of Modified IGBT Model

The Fourier-series-solution IGBT model described in [1] is modified according to model calculation sequence (8) using Johnson formula (13) to calculate \( \rho_{x2T} \).

As described above, the expected areas of improvement are:

1. The carrier distribution predicted by Fourier series solution does not drop to zero at MOS end, but converges to a non-zero value close to the one predicted by finite element simulation.
2. The forward drop is more accurately predicted by the model. The problem of an unrealistically large voltage drop at the MOS end is eliminated.
3. The problem of an unrealistically large voltage tail duration at turn-on is eliminated.

Results illustrating these three areas of improvement are now presented. Fig. 11 shows the steady-state carrier profile predicted by the modified model and Fig. 12 shows the carrier profile according to the finite element simulation. A fairly good agreement between the two carrier profiles is observed.

In order to validate the dependence on JFET cell geometry, simulations are performed for four different values of aspect ratio \( l_m/W_F \). The intercell half-length \( l_m \) is varied while keeping \( W_F \) and \( L \) unchanged. The results shown in Fig. 10 show a fairly good agreement.

<table>
<thead>
<tr>
<th>( l_m )</th>
<th>( W_F )</th>
<th>( L )</th>
</tr>
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<tbody>
<tr>
<td>11 ( \mu )m</td>
<td>10 ( \mu )m</td>
<td>17 ( \mu )m</td>
</tr>
<tr>
<td>( A )</td>
<td>( N_B )</td>
<td>( W_B )</td>
</tr>
<tr>
<td>1 cm(^2)</td>
<td>5E13 cm(^{-3})</td>
<td>266 ( \mu )m</td>
</tr>
</tbody>
</table>

Table 1: List of parameters used for 1.7kV Dynex IGBT
In order to validate the forward drop predictions of the modified model, a 1.7kV Dynex NPT IGBT is used. This IGBT has a cylindrical cell structure, which is transformed into a linear structure using the approach in [7]. The results in Table II show an excellent agreement between measured forward drop and model predictions. The experimental forward characteristic at 300K is shown in Fig. 13 and the predicted forward characteristic is shown in Fig. 14. Notice the excellent agreement of the fully-on forward characteristic (the one closest to the y-axis).

<table>
<thead>
<tr>
<th>Temperature</th>
<th>150K</th>
<th>300K</th>
<th>400K</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modified IGBT Model</strong></td>
<td>1.86V</td>
<td>2.63V</td>
<td>3.20V</td>
</tr>
<tr>
<td><strong>Experiment</strong></td>
<td>1.95V</td>
<td>2.64V</td>
<td>3.30V</td>
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<tr>
<td><strong>Silvaco</strong></td>
<td>-</td>
<td>2.90V</td>
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</tr>
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</table>

Table II: Forward drop at 100A/cm² for 1.7kV Dynex IGBT

To illustrate the improvement in voltage tail during resistive turn-on, experimental and simulation results are shown in Fig. 15 for the original IGBT model and in Fig. 16 for the modified model. Only the portion of the collector-emitter voltage below 100V is shown, so that the voltage tail is clearly visible. In the original model the voltage tail lasts more than 1.2µs, whereas in the modified model it lasts approximately 0.4µs and appears to be in much better agreement with the experimental waveform.

Notice that, strictly speaking, the intercell (JFET) carrier distribution used to calculate carrier concentration \( p_{\text{JT}} \) under the gate overlap is a steady-state solution only, not necessarily valid under transient conditions. However, the results in Fig. 15 and Fig. 16 clearly show an improvement in the transient response prediction of the model. This is probably due to the fact that the IGBT dynamic behavior is dominated by the slower carrier dynamics in the wide drift region and that the faster carrier dynamics in the small intercell region are well-approximated by a quasi-static description. This is similar to the modeling of the buffer layer in a PT IGBT, where a steady-state description of the buffer layer has proved to be adequate for transient conditions as well [2, 9].
results. Secondly, the forward drop predicted by the model is in good agreement with experimental results over a wide temperature range from 150K to 400K. Finally, the model has improved accuracy under transient conditions: the voltage tail at turn-on shows better agreement with experimental results than the original model.

This paper has examined lateral-gate IGBTs only. Extension of the results to trench-gate IGBTs [6] is left as future work.

IX. ACKNOWLEDGMENT

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