

Electric Machine Goodness: Classic results revisited in light of contemporary machine design

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Abstract — Common understanding among machine designers is that, when scaling down to smaller sizes, most performance parameters will degrade. This principle is embodied in the goodness factor presented by Laithwaite. A derivation of the goodness factor for axial flux geometry is presented, confirming that the result is the same as for radial flux geometry. The paper then discusses the factor with respect to permanent magnet machines, where experience shows that the principle does not hold, in that small machines can have very good characteristics. An initial modification of the factor to cover permanent magnet machines is presented.

1. Introduction

Laithwaite [1], in a classic paper, presents a goodness factor as a figure of merit for rotating machines. The basic idea is that the machine is an interconnection between an electric circuit and a magnetic circuit. The best electric circuit gives the greatest current for a given voltage (i.e., has the lowest resistance), and the best magnetic circuit gives the greatest flux for the a given mmf (i.e., has the lowest reluctance); thus, machine goodness must be inversely proportional to the product of resistance times reluctance. The "goodness factor" G is:

$$G = \frac{\omega}{\text{resistance} \times \text{reluctance}} = \frac{\omega \mu \sigma A_m A_e}{\ell_m \ell_e} \quad (1)$$

A indicates area and ℓ indicates length, whereas the subscripts m and e refer to magnetic and electric circuit, respectively. Permeability is denoted by μ and conductivity by σ .

Angular frequency ω is introduced to normalize the factor with respect to time. This gives a completely dimensionless factor.

The goodness factor is shown to tend to increase as the machine size increases, thus providing some

explanation for why large machine performance is often better than small machine performance.

The paper will derive expressions for goodness for several machine geometries, with and without permanent magnets, shedding considerable light on the reasons why axial-flux motors are good candidates for vehicular propulsion [3]-[5].

2. Conventional Induction Motor

The analysis of the goodness of the radial flux induction motor is straightforward. Let the pole pitch arc length be p [m] = pole pitch angle [rad] times radius [m], and the active length be ℓ [m]. Since the air gap length is relatively long, the magnetic circuit is dominated by the reluctance of the air gap. Thus, for the magnetic circuit, the cross-sectional area is $A_m \propto p \ell$, and its length is g , the air-gap length [m]. The electric circuit of the rotor has a length of ℓ (ignoring the end turns) and a cross-sectional area of $A_e \propto p t$, where t is the effective rotor conductor thickness [m]. Thus:

$$G \propto \frac{\omega \mu_0 \sigma p \ell p t}{g \ell} = \frac{\omega \mu_0 p^2}{\rho g / t} \quad (2)$$

where ρ is the rotor conductor resistivity [ohm-m]. The rotor conductor can be replaced by an equivalent surface resistivity of $\rho_r = \rho/t$ and the goodness becomes:

$$G \propto \frac{\omega \mu_0 p^2}{\rho_r g} \quad (3)$$

So, indeed the goodness is proportional to the square of the pole pitch and inversely proportional to the air-gap length g . Strictly speaking, the above equation applies to the rotor of the induction motor.

The original expression (1) makes it obvious that this goodness factor can also be considered as the ratio of X/R where X is the magnetizing reactance. It is obvious that machines with large pole pitch arc length p will have a large goodness factor. Although g will increase as the machine size increases, the amount of the increase is very small. This partly explains why large machines tend to have better performance in many respects than small machines. Economics and marketing also play a role, but we do not consider these factors in this paper.

Some empirical evidence to support these ideas is obtained by plotting the power rating per unit mass or power rating per unit volume versus power rating.

Figures 1 and 2 show the result for one US manufacturer's induction motors (all four-pole, 60-Hz, totally enclosed, blower-cooled induction motors). In both cases, the trend is obvious: larger kW sizes have better performance. Of course, these induction motors are typically applied in industrial locations where size and mass are usually of secondary consideration. In other applications, such as traction, size and weight are of primary importance. It is instructive that real induction machines do generally track the goodness factor, which is derived under very idealized assumptions.

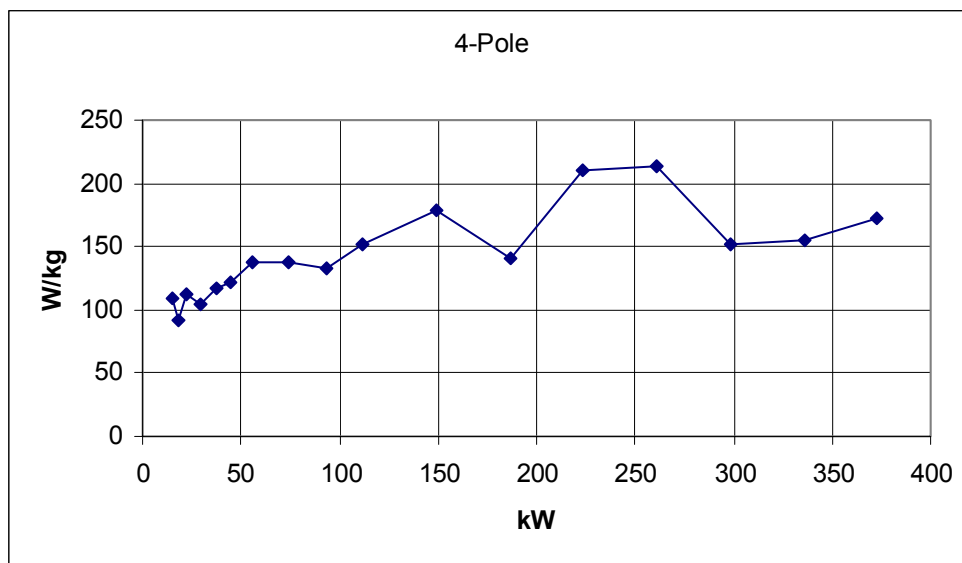


Fig .1. Typical induction motor power rating per unit mass [kW/kg] versus power rating [kW]

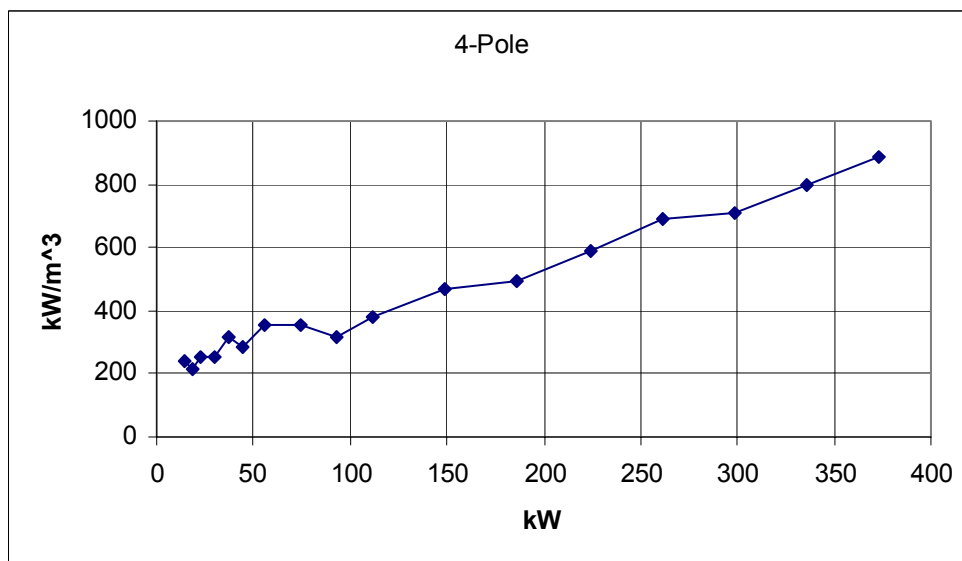


Fig .2. Typical induction motor power rating per unit volume [kW/m³] versus power rating [kW]

3. Axial Flux Geometry

Next, consider how the preceding analysis would change if the machine geometry is an axial flux configuration. The general expression (1) still applies. The assumed geometry is shown in Figure 3.

Magnetic circuit area and length and electric circuit area and length are given below:

$$A_m = \frac{\pi}{N_p} (R_o^2 - R_i^2)$$

$$\ell_m = g$$

$$A_e = \frac{2\pi (R_o + R_i) t}{N_p}$$

$$\ell_e = R_o - R_i$$

and pole pitch p is approximately the arc length at the average radius

$$p = \frac{2\pi (R_o + R_i)}{N_p}$$

so that

$$A_e = pt$$

and

$$A_m = \frac{p}{R_o + R_i} (R_o^2 - R_i^2) = p(R_o - R_i)$$

$$G \propto \frac{\omega \mu_o \sigma A_m A_e}{\ell_m \ell_e} = \frac{\omega \mu_o \sigma p^2 t (R_o - R_i)}{g (R_o - R_i)}$$

$$G \propto \frac{\omega \mu_o \sigma p^2 t}{g} \quad (4)$$

Interestingly, this is the same goodness factor that was previously derived for the radial flux machine. The same conclusion also applies, namely that large pole pitch p is good. Note that p is the average radius of the active region divided by the number of poles. This implies that similar scaling will be seen in the axial-flux machine as in the radial-flux machine. However, the machine designer may have more freedom in the axial-flux machine to make the pole pitch large, perhaps giving relatively better performance than a similar radial-flux machine.

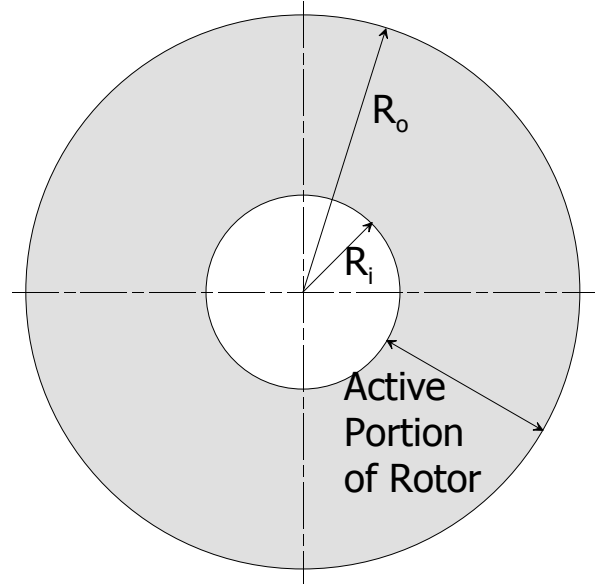


Fig.3. Axial flux machine geometry. Electric current is radial; flux is axial (perpendicular to plane of figure).

4. Permanent-Magnet Motors

The preceding analysis is concerned with induction motors, but the permanent-magnet motor is becoming very popular to the decrease in cost of good magnet materials, such as the rare-earth magnets (Nd-Fe-B, for example). These magnets are well known [6] to have large values of remanence B_r and coercivity H_c , with recoil permeability very close to μ_o . If the magnet length is ℓ and the air-gap length is g :

$$B = \frac{\ell}{g} B_r \quad (5)$$

Now the goodness factor, if it is to be a dimensionless quantity, must still have the same form, where the mmf is now the equivalent magnet mmf $\frac{\ell B_r}{\mu_o}$ and

$$\phi = B A = \frac{\ell A}{g} B_r \text{ so that the goodness becomes:}$$

$$G \propto B_r \frac{\ell}{g} A_m \frac{\mu_o}{B_r \ell} \frac{\sigma A_e}{\ell_e} \omega = \frac{\omega \mu_o \sigma A_m A_e}{g \ell_e} \quad (6)$$

which is the same expression as before.

There is, however, a fundamental question that has no clear answer: Could the goodness of the permanent-magnet motor be better defined by some ratio that is not dimensionless? In essence, the goodness of the magnets is solely related to flux that is produced, and there is no circuit setting up the flux. So one can question if it is fair to divide by the

equivalent magnet mmf just to obtain a normalized expression for goodness.

On balance, it seems that a goodness factor can be developed for permanent-magnet motors but its utility is somewhat questionable.

5. Experience with Axial Flux Permanent-Magnet Motors

In any case, the rare-earth magnet materials give us an extraordinary opportunity to design small motors with magnets on the rotor surface. The likelihood of demagnetization is very small, since the magnet has a relative permeability of approximately 1.

An axial flux machine that is designed primarily for high energy efficiency, as is typical of electric vehicular propulsion [3], may have a power/mass ratio that is an order of magnitude greater than the motors shown in Figure 1. The fundamental reasons are (1) design for high efficiency drives the designer to explore every option to reduce losses, giving a higher output torque than would be expected otherwise, (2) the availability of very good magnet materials, and (3) the geometry is more favorable.

To explore the issue of geometry, consider that the radial flux design allows a larger pole pitch p to be realized by an increase in the average radius of the active region. The last point is clearly shown in figure 3., if R_i and R_o are both doubled, the arc length of a pole pitch p is doubled. This results in quadrupling the goodness factor.

The radial flux machine designer can only increase the pole pitch by increasing the radius of the stator and rotor (i.e. using a larger frame diameter). Indeed, the interior of the rotor is largely wasted space, which is avoided by use of the axial flux design.

6. Conclusions

The idea of machine goodness, first described by Laithwaite [1] is explored for conventional machines and for machines with a radial flux design. The idea holds quite nicely for the induction motor, but is difficult to apply to the case of the permanent-magnet motor. In any case, it is obvious that the geometry of axial-flux machines is quite well suited to the development of motors with much greater volumetric efficiency than exhibited by conventional motors. The reasons are partly due to geometry and partly due to the primary design objectives.

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