Synergetic Control for Power Electronics Applications: a Comparison with the Sliding Mode Approach

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Abstract—The theory of synergetic control is introduced in a power electronics context in a previous paper. In this paper we review the theory, then we focus on a comparison with the sliding mode approach. Common elements and main differences are underlined and illustrated through a comparative simulation example. The main advantages of synergetic control are that it is well-suited for digital implementation, it gives constant switching frequency operation, and it gives better control of the off-manifold dynamics. Finally, simulation and experimental results under transient conditions are compared. Advanced control laws with adaptation are presented and discussed to show how to better exploit the features of the synergetic control. The example used throughout the paper is the control of a Boost converter operating in continuous conduction mode.

Keywords—Synergetic control theory, switching converter, nonlinear control, sliding mode control, dynamic parameter adaptation.

I. INTRODUCTION

Design of controllers for power converter systems presents interesting challenges. In the context of system theory, since power converters are non-linear time-varying systems, they represent a big challenge for control design.

Much effort has been spent to define small-signal linear approximations of power cells so that classical control theory could be applied to the design. See for example [1-3]. Those approaches make it possible to use a simple linear controller, e.g. Proportional-Integral (PI) controller, to stabilize the system. The most critical disadvantage is that the so-determined control is suited only for operation near a specific operating point. Further analyses are then necessary to determine the response characteristics under large signal variations [4-5].

Other design approaches try to overcome the problem by using the intrinsic non-linear and time-varying nature of switching converters for the control design purpose. A significant example of this approach is sliding mode control, used mostly for continuous-time systems [6-10]. This control theory has been extensively studied and applied to power electronics systems, since the variable-structure nature of power electronics systems allows a natural application of this theory. The most important advantages of this approach are order reduction, decoupling design procedure, and insensitivity to parameter changes. Disadvantages are the need of a fairly high bandwidth for the controller, which makes digital control solutions impractical, and chattering and variable switching frequency, which introduce undesirable noise in the system.

Alternative approaches suitable for discrete implementation include deadbeat control [11] and extensions of sliding mode control to discrete systems [12].

In this paper the focus is on a different approach, synergetic control [13-16], which tries to overcome the previously described problems of linear control by explicitly using a model of the system for control synthesis. The synergetic control shares with sliding mode control the properties of order reduction and decoupling design procedure, but it has several advantages. First of all, it is well suited for digital control implementation because it requires a fairly low bandwidth for the controller, but it requires comparatively more complex calculations than sliding mode, which can be easily realized digitally. A second advantage is that it operates at constant switching frequency and it does not have the chattering problems of sliding mode control, so that it causes less power filtering problems in power electronics applications.

The fact that synergetic control uses a model of the system for control synthesis can be considered both an advantage and a disadvantage. It appears desirable that the control uses all available information on the system for control purposes, but on the other hand it makes the control more sensitive to model and parameter errors. However, as we will demonstrate with experimental results, this problem can be solved.

This paper focuses on the application of synergetic control theory using a boost converter as the example application. In section II we review the general synergetic control design procedure. This procedure appears to have similarities with sliding mode control. Therefore, in section III we present a comparison between synergetic control and sliding mode, analyzing advantages and disadvantages of the two approaches. In section IV we apply the identified procedure

* The step load experimental and simulation results have been presented in [16].
to derive a control law for a boost converter. In section V we introduce a small-signal analysis of synergetic control which can be used for control design purposes. In section VI simulation results of synergetic control and sliding mode control are presented to demonstrate the advantages and disadvantages of the two approaches previously discussed. In section VII the step load problem in switching converters is briefly discussed and in section VIII experimental results of synergetic control are presented. An adaptive control technique is used to improve control performance: the approach solves a steady state error problem and significantly improves the control response speed.

II. SYNERGETIC CONTROL PROCEDURE

The synergetic control design procedure follows the Analytical Design of Aggregated Regulators (ADAR) method [14]. The main steps of the procedure can be summarized as follows.

Suppose the system to be controlled is described by a set of nonlinear differential equations of the form

$$\dot{x} = f(x, d, t)$$

(1)

where $x$ is the state vector, $d$ is the control input vector and $t$ is time.

Start by defining a macro-variable as a function of the state variables:

$$\psi = \psi(x)$$

(2)

The control will force the system to operate on the manifold $\psi = 0$. The designer can select the characteristics of this macro-variable according to the control specifications (e.g. limitation in the control output, and so on). In the trivial case the macro-variable can be a simple linear combination of the state variables.

The same process can be repeated, defining as many macro-variables as control channels.

The desired dynamic evolution of the macro-variable is

$$T \frac{d\psi}{dx} f(x, d, t) + \psi = 0$$

(3)

where $T$ is a design parameter specifying the convergence speed to the manifold specified by the macro-variable. The chain rule of differentiation gives

$$\psi = \frac{d\psi}{dx} \dot{x}$$

(4)

Combining (1), (3), (4) we obtain

$$T \frac{d\psi}{dx} f(x, d, t) + \psi = 0$$

(5)

Equation (5) is finally used to synthesize the control law $d$.

Summarizing, each manifold introduces a new constraint on the state space domain and reduces the order of the system, working in the direction of global stability.

The procedure summarized here can be easily implemented as a computer program for automatic synthesis of the control law or can be performed by hand for simple systems, such as the boost converter used for this study, which has a small number of state variables.

III. CHARACTERISTICS OF SYNERGETIC CONTROL AND COMPARISON WITH SLIDING MODE CONTROL

After describing the synergetic control procedure, it is appropriate to discuss some of its characteristics and compare it to sliding mode control, since there are similarities, but also significant differences.

Both synergetic control and sliding mode control have as their goal to force the system to operate on a manifold $\psi = 0$. This introduces an algebraic constraint on the system once it reaches the manifold, reducing its order. This is the so-called order reduction property. Notice that it is up to the designer to select a suitable manifold so that the new restricted system will have the required stability and dynamic characteristics.

Another property common to both control theories is the so-called decoupling property. The control problem can be broken up into two smaller problems, which can be solved separately, one related to the dynamics on the manifold, and the other related to the dynamics when off the manifold (the so-called hitting condition in sliding mode). For both controls, the manifold has to be appropriately chosen, so that the behavior of the reduced order system is satisfactory. If the same manifold is chosen for both controls, the dynamics on the manifold will be identical. A separate problem is how to choose the appropriate control action to force the system to reach the manifold and to stay on it. This is where sliding mode and synergetic control are different.

For switching converters, the control variable is the state $\delta$ of the active switch, which has two states, on (or 1) and off (or 0). Sliding mode basically operates as a bang-bang control around the manifold $\psi = 0$, according to the control law

$$\delta = 1 \text{ if } \psi < 0$$

(6)

$$\delta = 0 \text{ if } \psi > 0$$

Theoretically, this control law would give rise to infinite switching frequency. Usually, a hysteresis band is used to obtain a finite (but variable) switching frequency. On the
other hand, synergetic control applies to the converter the duty cycle \( d \) needed to enforce equation (3). Based on this difference, the following advantages of synergetic control can be identified:

- **Constant switching frequency.** Since synergetic control uses a PWM modulator, it provides a constant switching frequency, whereas sliding mode control naturally gives variable switching frequency.
- **Very suitable for digital implementation.** Enforcing control laws (6-7) for sliding mode control requires a fairly high bandwidth in the controller, which has to calculate the value of \( \psi \) with a bandwidth significantly higher than the (variable) switching frequency, to avoid delays in the switching action that would move the system state away from the manifold \( \psi = 0 \). The high bandwidth requirement makes sliding mode impractical for digital implementation. On the other hand, synergetic control calculates duty cycle \( d \) directly and therefore the bandwidth requirements are significantly less stringent. Typically, duty cycle need be calculated no more frequently than once per switching period so the more complex calculations typically needed for synergetic control can be easily accommodated with digital control.
- **Less sensitive to high-frequency noise.** Given the low bandwidth of the controller, less sensitivity to high frequency noise is expected.
- **Better controlled dynamics off the manifold.** As explained above, sliding mode control uses a bang-bang approach to force the system to reach the \( \psi = 0 \) manifold. The speed of convergence is not controlled in any way and depends solely on the original system dynamics with switch always on or always off. On the other hand, synergetic control imposes a well-controlled dynamic behavior off the manifold according to equation (3). The speed of convergence can be controlled using the time constant parameter \( T \) in equation (3).
- **The digital implementation makes the introduction of adaptation in the control algorithm much easier than in a conventional analog implementation.**

An interesting question that arises when comparing these two approaches is the following. Suppose we have a control manifold \( \psi = 0 \) that gives acceptable performance using one of the two techniques. Can the other technique be used as well? Or, are there any manifolds that work properly with one approach, but not with the other? In order to answer this question, we need to examine what it means to have an acceptable manifold. In the case of sliding mode, when selecting a control manifold \( \psi \), the designer has to consider three requirements: hitting, existence, and stability. The hitting question is whether the system, starting off the manifold and controlled according to (6-7), will move toward the manifold and eventually reach it. The existence question is whether a sliding mode exists on the manifold \( \psi = 0 \). The stability question is whether the reduced order system operating on the manifold \( \psi = 0 \) is stable and has satisfactory dynamics.

Similar questions must be answered for synergetic control. Will synergetic control make the system operate according to equation (3)? Once the system hits the manifold, will it stay on it? Does the reduced order system operating on the manifold have the desired stability and performance properties? Clearly the stability question is exactly the same for the two approaches, since the reduced order system obeys identical laws. Regarding the existence condition, it can be easily shown that under reasonable assumptions (continuity with respect to the control variable) the two approaches are equivalent. If a sliding mode exists, a synergetic control can be synthesized to keep the system on the \( \psi = 0 \) manifold, and, vice versa, if a synergetic control exists on a manifold, a sliding mode control exists as well. Regarding the hitting condition, we can say the following: if a system satisfies the hitting condition for sliding mode in a certain point, there exists a value \( T > 0 \) such that equation (3) is satisfied by an appropriate synergetic controller. On the other hand, if a synergetic controller that satisfies equation (3) can be found, that does not necessarily imply that in that point the hitting condition is satisfied. Therefore, synergetic control may be applicable in cases where sliding mode control cannot be used. The authors feel that in most cases of interest, if one approach can be used, the other one can be used as well.

As explained above, synergetic control uses a model of the system to be controlled in order to synthesize the control law. This approach follows the *internal-model principle*, which says that a good controller contains a model of the controlled system [17]. However, this characteristic can be considered a disadvantage, because:

- **Synergetic control needs a complete system model.**
- **Synergetic control tends to be more sensitive to discrepancies between parameters in the model and in the physical system.**

One obvious solution to the sensitivity problem is the adoption of sophisticated observers for parameter determination. This solution is reasonable only if the cost of the control is not a significant concern (e.g. high-power or high voltage applications). For situations where the control costs are of concern, we will show that suitable selection of the control macro-variables can largely resolve any sensitivity to uncertainty in system parameters. The example of the control of a Boost converter will be used to illustrate the issues involved in synergetic control implementation.
IV. THE BOOST CONVERTER CASE: AN EXAMPLE OF CONTROL SYNTHESIS

Let us consider the task of synthesizing a controller for a DC-DC boost converter (see Fig. 1). We will first describe the characteristics of the boost converter, then apply the general procedure described in the previous section.

In general, a dc-dc converter operating in continuous conduction mode can be represented by the following two sets of state equations

\[ \dot{x} = A_{\text{on}} x + B_{\text{on}} u \quad \text{switch on} \]  
\[ \dot{x} = A_{\text{off}} x + B_{\text{off}} u \quad \text{switch off} \]

where \( x \) is the state variable vector and \( u \) is the input variable vector, which includes the input voltage \( v_g \) and possibly load current \( i_0 \). Applying the state space averaging method [1], the averaged converter equations are

\[ \dot{x} = A(d)x + B(d)u \]  

where \( d \) is the duty cycle and

\[ A(d) = A_{\text{on}}d + A_{\text{off}}(1-d) \]  
\[ B(d) = B_{\text{on}}d + B_{\text{off}}(1-d) \]  

For the case of a Boost converter, the state-space-averaged equations are

\[ \begin{align*}
\dot{x}_1 &= -\frac{x_2}{L}(1-d) + \frac{1}{L}v_g, \\
\dot{x}_2 &= \frac{x_1}{C}(1-d) - \frac{x_2}{RC},
\end{align*} \]  

\[ 0 \leq d \leq 1, \]

where \( x_1 \) is the inductor current, \( x_2 \) the capacitor voltage and \( d \) the duty cycle.

Our objective is to obtain a control law \( d(x_1, x_2) \) as a function of state coordinates \( x_1, x_2 \), which provides the required values of converter output voltage \( x_2 = x_{2\text{ref}} \) and, therefore, current \( x_1 = x_{1\text{ref}} \) for various operating modes.

The limitation (12) must be satisfied. We use the procedure described above to solve the problem, i.e. to find \( d(x_1, x_2) \).

The first step is the choice of macro-variable. In general the macro-variable could be any function (including nonlinear functions) of the converter state. For the present time we will limit our investigation to a macro-variable that is a linear function of converter state, so that the system operates under a complete state feedback control. In order to force convergence to the desired steady state point, the macro-variable is chosen to be a linear combination of the state variable errors, so that the origin is the desired steady-state operating point. Therefore, the control has the general form

\[ \psi = K^T(x - x_{\text{ref}}) \]  

Without loss of generality, we choose \[ K^T = [k_1 \quad 1] \]

\[ \psi = k_1(x_1 - x_{1\text{ref}}) + x_2 - x_{2\text{ref}}. \]  

Substitution of \( \psi \) from (14) into the functional equation (3) yields

\[ T(\dot{x}_2 + k_1 \dot{x}_1) + (x_2 - x_{2\text{ref}}) + k_1(x_1 - x_{1\text{ref}}) = 0. \]  

Now substituting the derivatives \( \dot{x}_1(t) \) and \( \dot{x}_2(t) \) from (11) and solving for \( d \), the following control law is obtained:

\[ d = 1 - \left[ \frac{k_1}{L}v_g - \frac{x_2}{RC} + \frac{x_2 - x_{2\text{ref}}}{T} + k_1 \frac{x_1 - x_{1\text{ref}}}{T} \right] \left[ \frac{k_1}{L}x_2 - \frac{x_1}{C} \right] \]  

The expression for \( d \) is the desired control action for the converter. Notice that equation (16a) is different from the so-called equivalent control \( d_{\text{eq}} \) for sliding mode obtained using macro-variable (14), which is...
Notice that expression (16a) contains two extra terms, namely

\[
\frac{x_2 - x_{2\text{ref}}}{T} + k_1 \frac{x_1 - x_{1\text{ref}}}{T}
\]

Notice also that these extra terms are zero when the trajectory is on the manifold. As discussed in section III, the dynamics of synergetic and sliding mode on the manifold are identical, but they are different off the manifold. Equation (16a) is valid also off the manifold, whereas equivalent control equation (16b) is valid only on the manifold.

Control law (16) forces the state variable trajectory to satisfy equation (3). According to this equation, the trajectory converges to manifold \( \psi = 0 \) with a time constant \( T \) and then stays on the manifold \( \psi = 0 \) at all future times. So, from this point on, the state trajectory satisfies the equation

\[
\psi = k_i (x_1 - x_{1\text{ref}}) + x_2 - x_{2\text{ref}} = 0. \tag{17}
\]

This equation establishes a linear dependence between the two state variables \( x_1 \) and \( x_2 \), thereby reducing by one the order of the system. Moving on this manifold the trajectory eventually converges to the desired steady state: \( x_1 = x_{1\text{ref}} \); \( x_2 = x_{2\text{ref}} \). A geometric interpretation of the control law in the phase plane is shown in Fig. 2. The steady-state operating point is the origin, where the error goes to zero. Control equation (17) represents a straight line through the origin with slope \(-1/k\). The system operating point converges to the straight line (the manifold) and then moves along it to the origin.

V. SMALL-SIGNAL ANALYSIS OF SYNERGETIC CONTROL

In this section the procedure for a small-signal analysis for a converter under synergetic control is described. In the analysis it is assumed that the converter state is on the manifold and therefore equation (17) is satisfied. Under these assumptions, the equations describing synergetic control are identical to the equations for sliding mode control, if a sliding surface (17) is selected. Therefore, this analysis goes along the same lines of the small-signal analysis for sliding mode control described in [18].

The first step is to perturb and linearize the converter averaged equations (8) around an operating point. In this derivation the following convention is followed: capital variables represent operating point values and hat quantities (e.g. \( \hat{d} \)) represent small-signal perturbations. The operating point can be identified by the set of variables \((X, D, U)\), where the three variables are the operating point state, duty cycle, and input, respectively.

Following the state-space averaging procedure [1], we perturb equation (8) around the operating point \((X, D, U)\), obtaining

\[
\dot{\hat{x}} = A \hat{x} + B \hat{u} + B_d \hat{d} \tag{18}
\]

where

\[
A = A(D) \quad B = B(D) \tag{19}
\]

\[
B_d = (A_i - A_o)X + (B_i - B_o)U
\]

where \(A(D)\) and \(B(D)\) are given by equations (9), (10).

Equations (18) represent the small-signal linearized model for the converter.

The next step is to perturb manifold equation (13), which is linear. This gives

\[
K^T \dot{\hat{x}} = 0 \tag{20}
\]

Notice that it is assumed that the system operates on the manifold, and therefore it is \( \psi = 0 \). Since equation (20) is identically zero, its derivative is also zero

\[
K^T \dot{\hat{x}} = 0 \tag{21}
\]
Substituting equation (18) into equation (21) and solving for the small-signal duty cycle gives

$$\hat{d} = -(K^T B_d)^{-1} \left( K^T A \hat{x} + K^T B \hat{u} \right)$$  \hspace{1cm} (22)

This equation represents the small-signal duty cycle calculated by the synergetic control. In order to obtain the small-signal closed-loop system equations we can substitute the duty cycle from (22) back into (18), obtaining

$$\dot{x} = (A - B_d(K^T B_d)^{-1} K^T A) \hat{x} + (B - B_d(K^T B_d)^{-1} K^T B) \hat{u}$$  \hspace{1cm} (23)

Notice that $K^T B_d$ is a scalar quantity. Notice also that the small-signal effect of the synergetic control can be interpreted as a state feedback [17] with a feedback matrix

$$L = (K^T B_d)^{-1} K^T A$$

From (23) the small-signal closed-loop poles of the system are the eigenvalues of the system matrix

$$A_{eq} = A - B_d(K^T B_d)^{-1} K^T A$$  \hspace{1cm} (24)

Notice that synergetic control reduces by one the order of the system. This happens because manifold equation (20) represents a linear constraint among state variables. The effect on the closed-loop eigenvalues of (24) is that one eigenvalue is moved to the origin. This represents movement in the direction orthogonal to the manifold. Since the initial condition puts the state on the manifold and the eigenvalue for movement in a direction orthogonal to the manifold is zero, the system will stay on the manifold. It is possible to complete the small-signal model derivation and obtain a reduced order closed-loop system, by combining (20) and (23) as explained in [18].

VI. SYNERGETIC CONTROL AND SLIDING MODE CONTROL: SIMULATION RESULTS

At this point it is of interest to compare simulation results of sliding mode control and synergetic control for the Boost converter case. The differences between the two methods described above are clearly exemplified by the simulation.

A startup transient is simulated using the Virtual Test Bed (VTB) software developed by the University of South Carolina [19]. The same manifold is used for both control methods, so that, once on the manifold, the dynamics are identical. A first difference worth noticing is that synergetic control simulation can be performed using either a switching model or an averaged model for the converter, whereas the sliding mode simulation can only be performed by using a switching model.

Design parameters of the Boost converter are: filter inductance $L = 46 \, \mu\text{H}$, output capacitance $C = 1360 \, \mu\text{F}$, load resistance $R = 35 \, \Omega$, input voltage $V_p = 12 \, \text{V}$ and desired output voltage $40\, \text{V}$. The switching frequency for synergetic control is 50kHz and the hysteresis band of sliding mode control is adjusted to obtain approximately the same switching frequency. The manifold is the same for the two controls and is given by (14) with $x_{1\text{ref}} = 3.81 \, \text{A}$, $x_{2\text{ref}} = 40 \, \text{V}$, $k_1 = 1$. The value of $x_{2\text{ref}}$ is chosen based on the nominal load resistance. Parameter $k_1$ determines the dynamic behavior of the converter. Its choice is dictated by the need to ensure stability, reasonable transient response time and limit inductor current $x_1$ during transient. The small-signal analysis of section V shows that the closed-loop time constant decreases as $k_1$ decreases and that for $k_1 = 1.6E-3$ the system goes unstable. Therefore this is a lower bound on the value of $k_1$. A small value of $k_1$ also causes a large transient inductor current. At startup the inductor current will reach its maximum value when the converter state first hits the manifold. If at startup the output voltage error is $\Delta v_0$, it is clear from Fig. 2 that an upper bound for the inductor current is given by $i_{1\text{ref}} + \Delta v_0/k_1$. To limit the inductor current, the value $k_1 = 1$ is chosen. From the small-signal analysis the system closed-loop time constant in steady state is $T_d = 2\, \text{ms}$. More complicated control laws could be used to limit transient inductor current and are described in [15], but they will not be used in this example, since its purpose is just to demonstrate features of sliding mode and synergetic control. Finally for the synergetic control the time constant $T$ in equation (3) is set at $T = 0.3 \, \text{ms}$, about an order of magnitude smaller than the closed loop small-signal time constant. This ensures that at startup the converter state will reach the manifold in approximately 1ms.

The schematic for the sliding mode simulation is shown in Fig. 3. The sliding mode control with the hysteresis band is clearly identifiable in the lower half of the figure. Figs. 4, 5, 6 show the output voltage transient, the inductor current transient and the phase portrait, respectively. Notice that at the beginning, when the converter state is off the manifold with $\psi < 0$, the switch is always closed according to (6) and the inductor current reaches a peak value of about 30 A. Since the initial voltage error is $\Delta v_0 = 25 \, \text{V}$, this value is in good agreement with the expected value of $i_{1\text{ref}} + \Delta v_0/k_1 = 28.81 \, \text{A}$. After reaching the manifold, the motion along the manifold is clearly visible from Fig. 6.

The synergetic control simulation results for the same variables are shown in Figs. 7-9. The output voltage waveform is similar. Notice that the peak value of the inductor current is only 23A, as compared with 30A for sliding mode, and that before the manifold is reached, the duty cycle is not unity. This demonstrates the fact that synergetic control gives a better control of the off-manifold dynamics.
Using synergetic control it is also possible to perform simulations using averaged converter models, with advantages in terms of simulation speed and waveform readability, since switching frequency ripple is eliminated. Figs. 10-11 show the output voltage and the phase portrait for an averaged model simulation of synergetic control. The well-controlled convergence to the manifold in Fig. 11 should be compared with the abrupt bang-bang-type approach of sliding mode in Fig. 6. Fig. 11 shows the phase portrait for various values of parameter $T$, showing that increasing $T$ gives a more gentle approach to the manifold. In effect the sliding mode will operate with maximum or minimum duty cycle, limiting the time needed to hit the manifold. For the synergetic control this process can be tuned by choosing different values for the main time constant $T$ introduced in equation (3). If we accept some delay in this process, we can limit the maximum current to any specified value.

On the other hand, the simulation with switching converter model shows also the different level of sensitivity to parameter variations typical of these two control structures. While the sliding mode control perfectly reaches the right steady-state condition of 40 V, the synergetic control has some steady state error. This can be justified considering that the model we used to synthesize the control law does not include any voltage drop across the silicon components. In the final part of the paper the authors will show how this limitation can be easily overcome with adaptive control laws.

From the last two figures we can appreciate another characteristic of the synergetic control. Considering that the control synthesis is based on the averaged model, an averaged-model-based simulation can be performed. This can speed up the testing process significantly by reducing the computational time. It is also interesting to observe that, since in this case the switch is ideal, the voltage perfectly reaches 40 V as set by the reference.

![Figure 3. Schematic for sliding mode simulation in VTB.](image-url)
Figure 4. Output voltage under sliding mode control. Simulation with switching converter model.

Figure 5. Inductor current under sliding mode control. Simulation with switching converter model.

Figure 6. Phase portrait under sliding mode control. Simulation with switching converter model.

Figure 7. Output voltage under synergetic control. Simulation with switching converter model.

Figure 8. Inductor current under synergetic control. Simulation with switching converter model.

Figure 9. Phase portrait under synergetic control. Simulation with switching converter model.
VII. STEP-LOAD VARIATION PROBLEM

Analyzing the procedure reported above, a weakness in the procedure could be easily identified: it theoretically requires precise knowledge of the model parameters. This requirement is a serious limitation for two reasons:

- Sometimes it is not easy to identify the parameters
- Sometimes the parameters change in time

In this paper we focus mostly on the second problem considering a situation where the load changes suddenly from one value of resistance to another.

This problem seems to be particularly interesting for at least two reasons:

1. It is a typical problem for power electronics, where the power supply is supposed to compensate quickly for load variation.
2. It is an experiment that can be easily performed, both in simulation and in the laboratory, just by switching on and off a resistor bank.

Designing model observers that are able to catch the variation of the parameters and adapt the control accordingly, could solve the problem. However, this solution will not be considered in this case: we focus instead on the control theory itself, striving to select a robust manifold. The observer solution can be applied as a second step to improve the performance of a control structure that is already robust. An advantage of this approach is that, if we can find a solution that just modifies the manifold definition without adding an observer, we can limit the computational burden of the controller implementation.

In the following, different definitions of the control law are considered and both simulation and experimental results are given.

VIII. SIMULATION AND EXPERIMENTAL RESULTS

An extended simulation analysis was conducted to verify the control performance. The simulations were performed both in the Matlab environment and using the VTB simulator [19]. After the theoretical analysis, a laboratory prototype was designed and built. The synergetic control is well suited for a digital implementation, so a DSP-based platform was selected for the migration from the VTB environment to the real world.

The laboratory prototype has the following component values:

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Input Voltage:</td>
<td>12 V</td>
</tr>
<tr>
<td>Input Inductance:</td>
<td>L = 46 μH</td>
</tr>
<tr>
<td>Rated Output Voltage:</td>
<td>40 V</td>
</tr>
<tr>
<td>Output Capacitance:</td>
<td>1360 μF</td>
</tr>
<tr>
<td>Maximum Load:</td>
<td>50 W</td>
</tr>
<tr>
<td>Main Switch:</td>
<td>IRF540N</td>
</tr>
<tr>
<td>Switching Frequency:</td>
<td>50kHz</td>
</tr>
</tbody>
</table>

The sampling rate of the DSP has been chosen to be approximately equal to the switching frequency. Appropriate analog anti-aliasing filters are used for the analog variables fed back to the digital control. The DSP provides a continuous control variable representing the duty cycle, which is synthesized by an analog PWM circuit. Therefore there is no need to synchronize the DSP with the switching frequency. Notice also that the synergetic control law has the form of (16a) and therefore it is a purely algebraic law (with the possible exception of the highpass filtered version described...
below), so the digital control is an instantaneous law with no state.

Several different control laws were tested under the condition of a step change of the load from 35 Ohms to 70 Ohms. The experimental and simulation results for the different control laws both show the startup transient first, followed by the response to step load variation. This was done to verify the large-signal stability of the system under the various control laws.

A. Simple Synergetic Control Law

As a first case we adopt the manifold definition of equation (14). This case is expected to be the least robust to load variation because equation (14) uses the current reference, which is a function of the load value.

Both in simulation and experiment, we see that the output voltage is disturbed by the step load variation, reaching a new steady-state value that is different from the desired 40 V reference (see Fig. 12). Beside the steady-state error, the system response to the step load variation is fairly slow and it is desirable to improve it.

B. Control Law with Highpass-Filtered Inductor Current

As noted above, one of the reasons for the steady-state output voltage error after the change of the load is the fact that the synergetic control law (14) incorporates the inductor current reference – but that reference cannot be a static quantity when the load is variable. A solution to this problem is to adopt a control law that uses a highpass-filtered measurement of the inductor current [18]

\[ \psi = x_2 - x_{2\text{ref}} + k_1 i_1 \frac{sT}{1 + sT} \]  

The result is shown in Fig. 13. Notice that the steady-state output voltage error is significantly reduced with respect to the simple case of Fig. 12. Still, this solution does not improve the response speed.

C. Adaptation of Control Parameter

Now we introduce a control law with adaptive gain [9]. With reference to equation (14), gain \( k_1 \) is varied according to the following expression:

\[ k_1 = \alpha + \beta |x_2 - x_{2\text{ref}}| \]  

Fig. 14 shows a geometric interpretation of this control law. Far from the origin the error is large and \( 1/k_1 \) is small. This situation is represented by the line with smaller slope. As the operating point moves closer to the origin the trajectory slope increases as \( 1/k_1 \) increases. The adaptation is a continuous process and the trajectory slope changes continuously. Only three representative lines are shown in the figure.
The motivation for the choice of this control law is that, when the output voltage error is large, a large value of $k_1$ is desirable to limit the inductor current. However, this implies a slower dynamic response. On the other hand, when the output voltage error is small, a small value of $k_1$ gives a faster dynamic response. This adaptive control law limits the inductor current during large transient, such as at startup, and it gives fast transient response when the output voltage error is small, such as during a step load variation. In this example, the adaptive control law uses the following parameter values: $\alpha = 0.03$, $\beta = 0.05$. The value of $\alpha$ was chosen to guarantee a good transient response for step load. Applying the small-signal analysis procedure of section V, the chosen value of $\alpha$ gives a small-signal closed-loop time constant $T_{cl} = 0.12\text{ms}$. The value of $\beta$ is chosen to limit the peak inductor current during startup. For a desired output voltage of 40 V with an input voltage of 12 V, at the beginning of the startup process the magnitude of the output voltage error is $|x_2 - x_{2,\text{ref}}| = 28\text{V}$. Adaptive law (27) gives a value of $k_1 = 1.43$. Therefore, the peak inductor current - which happens when the converter state first hits the manifold - is limited to $|x_2 - x_{2,\text{ref}}| k_1 = 19.6\text{A}$.

The results obtained with this approach are illustrated in Fig. 15. We see that introduction of the adaptive term has a positive effect on control performance. The startup transient speed is increased and the effect of step load change on the output voltage is so small that it is not discernible with the voltage scale used. This indicates that the steady-state error is reduced with respect to the simple case and the speed of response is significantly improved. Fig. 16 shows the inductor current waveform with the initial overshoot and the response to the step load change. Notice that the inductor current is limited to 20A as desired.

**Fig. 15: Output voltage (Simulation and experiment) results with adaptive control law**

<table>
<thead>
<tr>
<th>time(S)</th>
<th>volt(V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>0.04</td>
<td>20</td>
</tr>
<tr>
<td>0.06</td>
<td>25</td>
</tr>
<tr>
<td>0.08</td>
<td>30</td>
</tr>
<tr>
<td>0.1</td>
<td>35</td>
</tr>
</tbody>
</table>

**Fig. 16: Inductor current with adaptive control law**

**IX. DISCUSSION**

At this point we can reach some conclusion regarding the different control laws used. The simple control law exhibits a very slow response time and gives rise to a significant steady-state error. The step load response time can be estimated on the basis of the small-signal eigenvalue calculated from (24). The result for the simple law in Fig. 12 was obtained for $k_1 = 1$. For that value of $k_1$, equation (24) predicts a first order system with a time constant $T_{cl} = 4\text{ms}$. This is in very good agreement with the simulation and experimental results. Fig. 17 is a zoomed version of Fig. 12 showing the detail of the step load response. The response time is approximately 12ms, equal to $3T_{cl}$. Analysis using equation (24) shows that the response time could be reduced by reducing $k_1$. Unfortunately, simulation shows that the large-signal stability of the system is compromised by a choice of $k_1$ smaller than 0.5. The conclusion is that with the simple control law it is difficult to obtain large-signal stability and fast step load response.

The control law with highpass-filtered inductor current provides some improvements: it reduces the steady-state error, but it does not give an improvement in step response speed.

The adaptive control allows the achievement of these goals by reducing $k_1$ when the converter is close to steady-state, guaranteeing fast step load response without compromising large-signal stability. Fig. 18 is a zoomed version of Fig. 15 showing the detail of the step load response. The steady-state error is 0.2V, which is 0.5%. Notice also that the response time is approximately 0.8ms, which is 15 times faster than the response with the simple control law. This is in fairly good agreement with the estimation from equation (24). In this case the value of $k_1$ changes somewhat due to the variation in the output voltage, and it is estimated to be around 0.05. The closed-loop time constant predicted from equation (24) is $T_{cl} = 0.16\text{ms}$, which gives an estimated response time of 0.48ms.

In conclusion, the adaptive control gives the better trade-off between large-signal stability and load step response time.
The small-signal analysis allows to predict the step load response time and can be used for control design.

Fig. 17: Step load response for the simple synergetic control (zoomed version of Fig. 12)

Fig. 18: Step load response for the adaptive synergetic control (zoomed version of Fig. 15)

X. Conclusions

The paper discussed various issues regarding synergetic control as applied to switching power converters. First of all, synergetic control was introduced and compared with sliding mode control. Advantages and disadvantages were discussed and illustrated with a simulation example using a Boost converter. After that, simulation and experimental results for the Boost converter operating under synergetic control were presented. We analyzed different approaches to solving the problems introduced by step variations of the load. Although a high sensitivity to parameter variations was one of the most important concerns, and a potentially serious disadvantage of the synergetic approach, as compared to the sliding mode approach, our results showed that it is possible to synthesize a control law that is highly insensitive to parameter variations. Finally, we presented a small-signal analysis technique that can be used to help in the selection of design parameters such as gain $k_1$.

XI. Acknowledgment

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XII. References


