Synergetic Control for Dc-Dc Boost Converter: Implementation Options

E. Santi, A. Monti, D. Li, K. Proddutur, R. Dougal
Department of Electrical Engineering
University of South Carolina
Swearingen Engineering Center, Columbia, SC 29208 U.S.A

Abstract—The theory of synergetic control was introduced in a power electronics context in a previous paper. In this paper we review the theory, then focus on some practical aspects with reference to both simulations and actual hardware. In particular we address management of the current limit condition and solve it with several different approaches. Adaptive and other control laws are introduced to improve the control performance. Various other control laws are introduced to improve the control performance, and then they are evaluated by applying the classical voltage reference step test.

Keywords—Synergetic control theory, switching converter, nonlinear control, current limit, dynamic parameter adaptation, integral error term.

I. INTRODUCTION

This paper focuses on the application of Synergetic control theory using a boost converter as the example application.

The Synergetic control theory was introduced in general terms in [1]. Its application to switching power converters is discussed in [2]. The present work expands the results obtained in [2] by exploring different control options that improve the system performance.

In section II we review the general synergetic control design procedure. In section III this procedure is applied to a boost converter deriving a basic control law. In section IV simulation and experimental results for the control are presented. In section V various improved synergetic control laws are proposed and their performance verified by simulation and by experiment. The first case considered is the introduction of a current limit and two different control laws are proposed. The second case considered is a control law with parameter adaptation. It reduces steady-state output voltage error and provides faster response without increasing the peak inductor current. The third case introduces an integral voltage error term to eliminate the steady-state output voltage error. Finally a control law that does not need an inductor current reference is proposed.

II. SYNERGETIC CONTROL PROCEDURE

The synergetic control procedure consists of the following steps:

- Start by defining a macro-variable as a function of the state variables:

\[ \psi(t) = \psi(x,t) \]  

The control will force the system to operate on the manifold \( \psi = 0 \). The designer can select the characteristics of this macro-variable according to the control specifications (e.g. limitation in the control output, and so on). In the trivial case the macro-variable is a simple linear combination of the state variables.

- Repeat the same process defining as many macro-variables as control channels.

- Fix the dynamic evolution of the macro-variables according to the equation:

\[ T \dot{\psi}(t) + \psi = 0; \ T > 0 \]  

where \( T \) is a design parameter that specifies the convergence speed to the manifold \( \psi = 0 \) defined by the macro-variable.

- Synthesize the control law (evolution in time of the control output) according to equation (2) and the dynamic model of the system.

In summary, each manifold introduces a new constraint on the state space domain that reduces the order of the system, working in the direction of global stability.

The procedure summarized here can be easily implemented as a computer program for automatic synthesis of the control law or it can be performed by hand for simple systems that have a small number of state variables (such as the boost converter).

By suitable selection of macro-variables the designer can obtain interesting characteristics for the final system such as:

- Global stability
- Parameter insensitivity
- Noise suppression

The synergetic control theory guarantees the global stability on the manifold: this means that once we reach the manifold the system is not supposed to leave it even for large
signal variation. This condition ensures that the system will keep the reduced order characteristic but does not guarantee the global stability of the system itself. It is up to the designer to select a suitable manifold so that the new restricted system will have the required stability characteristics.

These results are obtained by working on the full nonlinear system; the designer does not need to introduce simplifications in the modeling process to obtain a linear description as is necessary for application of classical control theory.

III. THE BOOST CONVERTER CASE: AN EXAMPLE OF CONTROL SYNTHESIS

Let us consider the synthesis of a controller for a DC-DC boost converter (see Fig. 1).

The classical averaged model of the converter is:

\[
\begin{align*}
\dot{x}_1(t) &= -\frac{x_2}{L}(1-u) + \frac{1}{L}V_g, \\
\dot{x}_2(t) &= \frac{x_1}{C}(1-u) - \frac{x_2}{RC},
\end{align*}
\]  

where \( x_1 \) is the inductor current, \( x_2 \) the capacitor voltage and \( u \) the duty cycle.

The synthesis task is to obtain a control law \( u(x_1,x_2) \) as a function of state coordinates \( x_1, x_2 \), which provides the required values of converter output voltage \( x_2 = x_{2\text{ref}} \) and, consequently, current \( x_1 = x_{1\text{ref}} \) for all operating modes. The limitation (4) must be satisfied. We use the ADAR method [1] to find \( u(x_1,x_2) \). According to this method, we introduce the following macro-variable

\[
\psi = (x_2 - x_{2\text{ref}}) + k(x_1 - x_{1\text{ref}}).
\]  

Substituting \( \psi \) from equation (5) into the functional equation

\[
T\psi(t) + \psi = 0, \quad T > 0,
\]  

yields

\[
T(\dot{x}_2 + k\dot{x}_1) + (x_2 - x_{2\text{ref}}) + k(x_1 - x_{1\text{ref}}) = 0.
\]  

Now substituting the derivatives \( \dot{x}_1(t) \) and \( \dot{x}_2(t) \) from equations (3) and (4), the control law is obtained:

\[
u = 1 - \left[ \frac{k}{L} V_g - \frac{x_2}{RC} + \frac{x_2 - x_{2\text{ref}}}{T} + k\frac{x_1 - x_{1\text{ref}}}{T} \right] \left[ \frac{k}{L} x_2 - \frac{x_1}{C} \right].
\]  

This equation establishes a linear dependence between the two state variables \( x_1 \) and \( x_2 \), thereby reducing by one the order of the system. Moving on this manifold the trajectory eventually converges to the converter’s steady state: \( x_1 = x_{1\text{ref}}, \ x_2 = x_{2\text{ref}} \). A geometric interpretation of the control law in the phase plane is shown in Fig. 2. The steady-state operating point is the origin, where the error goes to zero. Control equation (9) represents a straight line through the origin with slope \(-1/k\). The system operating point converges to the straight line and then moves along it to the origin.

We note here that the actual law used in both simulation and experiments reported in the following is slightly more complex than this because we desired to account for non-idealities (voltage drops) in the power switches.
IV. SYSTEM SIMULATION AND EXPERIMENTAL VERIFICATION

An extended simulation analysis was conducted to verify the control performance. The simulations were performed both in the Matlab environment and using the VTB simulator [3]. After the theoretical analysis a laboratory prototype was designed and built. The Synergetic control is well suited for a digital implementation so a DSP-based platform was selected for the migration from the VTB environment to the real world.

The laboratory prototype has the following component values:

<table>
<thead>
<tr>
<th>Component Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Input Voltage</td>
<td>12 V</td>
</tr>
<tr>
<td>Input Inductance</td>
<td>L = 46 µH</td>
</tr>
<tr>
<td>Rated Output Voltage</td>
<td>40 V</td>
</tr>
<tr>
<td>Output Filter Capacitance</td>
<td>1360 µF</td>
</tr>
<tr>
<td>Maximum Load</td>
<td>100 W</td>
</tr>
<tr>
<td>Main Switch</td>
<td>IRF540N</td>
</tr>
</tbody>
</table>

A very good agreement was found between simulation and experiment. As an example, a voltage reference step response, from 20 to 40 V is shown in Fig. 3.

Fig. 4 shows the time evolution of the macro variable $\psi$. Notice the smaller time scale of 0.5ms per division. When the reference step is applied, the state trajectory momentarily leaves the $\psi = 0$ manifold, but in approximately 1ms goes back to the manifold. This is consistent with the value of $T=0.3\text{ms}$ used: as expected this transient decays in a time equal to $3T$. At this point we are approximately at time $t=0.2$s in Fig. 3. Once this transient is over, a second transient according to equation (9) follows. This transient lasts approximately 20ms. It is usually desirable to choose the time constant $T$ significantly shorter than the response time of the control, so that during most of the transient the system is on the $\psi = 0$ manifold.

The following observations can be made regarding the control response waveforms of Figs. 3-4. These observations provide motivation for the alternative control laws described in the rest of the paper:

- Current overshoot is not limited. The control law does not limit the inductor current. At startup and during large transients the inductor current can temporarily reach undesirably large values. In the next section two different implementations of a current limit are described.
- Macro-variable $\psi$ has a steady-state value different from zero. The reason for this (see [2]) is that the control law in reality enforces condition (6) and only indirectly condition (9). Therefore any discrepancy
between the real converter parameters and the ones assumed in the control law synthesis may lead to a non-zero value of $\psi$ in steady state.

- Steady state output voltage error. Careful examination of the experimental waveforms shows that a steady-state error is present. Before the step, there is a 0.5 V error and after the step there is a 0.15 V error. In general this error is caused by small discrepancies between the real converter parameters and the ones used for the control synthesis. The non-zero value of macro-variable $\psi$ is a symptom of the same problem. In order to reduce the error, one approach is to adapt the value of parameter $k$ in the control law (5) because a smaller value of $k$ reduces the voltage error due to the term $k(x_1 - x_{1\text{ref}})$ in equation (5). It should be noted that there is an uncertainty in the value of the current reference value $x_{1\text{ref}}$. Another approach is to add to the control law (5) an integral error term, so that in steady-state the error goes to zero.

- The synthesized control variable $u$ given by equation (8) requires knowledge of converter parameters such as inductance $L$, capacitance $C$ and also load resistance $R$. In particular load resistance $R$ is needed to calculate the reference inductor current $x_{1\text{ref}}$ used in the control law (5). Usually in a switching converter application it is reasonable to assume that inductance and capacitance values are known, but it is not reasonable to assume that the load characteristics are known or even fixed. So we introduce a modified control that does not use load resistance $R$, but rather uses a high pass-filtered version of inductor current in equation (5), so that the control does not need the reference inductor current $x_{1\text{ref}}$.

- There is a discrepancy in the steady-state inductor current between experiment and simulation, especially when the output voltage is 40 V. This discrepancy is due in part to the fact that the experimental voltage is higher than the simulated voltage as noted above. Another reason is that in the simulation the boost converter model is ideal and therefore loss less. In the experiment the losses in the converter will make the inductor current larger than the value predicted by the simulation.

V. SYNTHESIS OF MODIFIED / IMPROVED CONTROLS

The previous case illustrated a very simple case of control synthesis that transformed the boost circuit into a first order system always working on the manifold described by the macro-variable.

Areas of possible improvement were identified above. More complex macro-variable definitions will next be introduced to implement improved control laws. The procedure to derive the new control laws is analogous to the one described above. In the following part of this paper a number of different cases will be described in detail and their performance evaluated both in simulation and experiment.

A. Current limit implementation

One classical problem relates to imposing a limit on one of the state variables, e.g. limiting the maximum input current. Two different approaches are examined.

In one approach the macro-variable is defined as a piecewise linear function.

$$\psi_1 = (x_2 - x_{2\text{ref}}) + k (x_1 - x_{1\text{ref}}) \quad \text{for} \quad x_2 > x_{2\text{TH}} \quad (10)$$

$$\psi_1 = x_1 - x_{1\text{MAX}} \quad \text{for} \quad x_2 < x_{2\text{TH}} \quad (11)$$

where $x_{2\text{TH}}$ is the voltage value at which current $x_1$ is equal to the limit value $x_{1\text{MAX}}$. This value can be easily calculated from equation (9) as

$$x_{2\text{TH}} = x_{2\text{ref}} - k(x_{1\text{MAX}} - x_{1\text{ref}}) \quad (12)$$

A geometric interpretation of this control law is shown in Fig. 5, which shows the control law (10)-(11) in the phase plane. The steady-state operating point is the origin, where the error goes to zero. Control equation (10) represents a straight line through the origin with slope $-1/k$. The current limit given by equation (11) represents a straight horizontal line.

![Figure 5. Piecewise linear function in the phase plane represents the current limit control law.](image)

Repeating the usual synthesis procedure, the control law can be obtained:

$$u = 1 - \left[ k \frac{V_g - x_2}{L} + \frac{x_2 - x_{2\text{ref}}}{x_{1\text{MAX}} - x_{1\text{ref}}} + k \frac{x_1 - x_{1\text{ref}}}{T} \left( \frac{k x_2 - x_1}{C} \right) \right] \left( \frac{k x_2 - x_1}{x_{2\text{TH}}} \right) \quad \text{for} \quad x_2 > x_{2\text{TH}} \quad (13)$$

$$u = 1 - \left[ \frac{V_g + L}{T} (x_1 - x_{1\text{MAX}}) \right] / x_2 \quad \text{for} \quad x_2 < x_{2\text{TH}} \quad (14)$$
This approach is easy to implement with a digital controller. It has been simulated and tested and results are shown in Fig. 6. Once again note the good agreement between simulation and experiment.

A second possible approach to implement the current limit is to define a single macro-variable that includes the current limit in its definition. A possible control law is

\[
\psi_2 = x_1 + A \cdot \tanh \left( \frac{-x_{1\text{ref}} + (x_2 - x_{2\text{ref}}) / k}{A} \right) \tag{15}
\]

where \( A = x_{1\text{MAX}} \). This new definition will determine a new manifold where the current \( x_1 \) is naturally limited by the hyperbolic tangent function to the range \( \pm A \) whenever the system state is on the manifold. A geometric interpretation of the control law is shown in Fig. 7.

The control law can be easily obtained:

\[
u = 1 - \frac{\frac{x_2}{L} - \frac{x_{1\text{ref}}}{R} \tanh \left( \frac{-x_{1\text{ref}} + (x_2 - x_{2\text{ref}}) / k}{A} \right) + A \tan \left( \frac{-x_{1\text{ref}} + (x_2 - x_{2\text{ref}}) / k}{A} \right)}{\frac{x_2}{L} - \frac{x_{1\text{ref}}}{R} \tanh \left( \frac{-x_{1\text{ref}} + (x_2 - x_{2\text{ref}}) / k}{A} \right)} \tag{16}
\]

The simulation and experiment are shown in Fig. 8. An excellent agreement has been found between simulation and experiment.

B. Control Law with Dynamic Adaptation of Control Parameter

The choice of the value of control parameter \( k \) in the macro-variable definition (5) involves a trade-off: during transient a relatively large value of \( k \) is desirable, because it slows down the transient and avoids large overcurrents and excessive stress on the switches. On the other hand, a small value is desirable in steady state, because this reduces the steady state output voltage error as explained in [2]. A way around the trade-off is to dynamically adjust the value of \( k \) as a function of output voltage error, reducing \( k \) when the output voltage error is small. Based on this consideration, \( k \) can be chosen as follows:

\[
k = 0.03 + 0.05 \cdot \text{abs}(x_2 - x_{2\text{ref}}) \tag{17}
\]
Fig. 9 shows a geometric interpretation of this control law. Far from the origin the error is large and $1/k$ is small. This situation is represented by the line with smaller slope. As the operating point moves closer to the origin the trajectory slope increases as $1/k$ increases. The adaptation is a continuous process and the trajectory slope changes continuously. Only three representative lines are shown in the figure.

Using the above value of $k$ in the control law (5), the simulation and experiment are shown in Fig. 10. These results can be compared with those in Fig. 3, where $k = 1.0$. First of all, the steady-state error is reduced with respect to Fig. 3. An added benefit is that the final part of the transient, when the output voltage error is small and parameter $k$ is small according to equation (17), is faster with the modified control law for the same peak inductor current of approximately 20 A.

It is also possible to combine the parameter adaptation with the current limit feature introduced above by using the value of $k$ given by equation (17) in control law (15) that implements the current limit. The current limit was set at 10 A. The result is shown in Fig. 11. Notice that the inductor current is limited to less than 10 A.
C. Control Law with Integral Error Term

In order to eliminate the steady-state error an integral error term is added to the manifold definition. This term is amplitude-limited to avoid windup problem and interference with the synergetic control. The improved macro-variable is

$$\psi = (x_2 - x_{2,ref}) + k_1 (x_1 - x_{1,ref}) + k_2 \int (x_2 - x_{2,ref}) \, dt \quad (18)$$

According to the synthesis procedure described in section II, the control law is derived as

$$u = \frac{k}{L} \frac{V_s}{RC} \frac{x_2 - x_{2,ref}}{T} + \frac{k}{T} \frac{x_1 - x_{1,ref}}{T} + k_1 (x_1 - x_{1,ref}) + k_2 \int (x_2 - x_{2,ref}) \, dt$$

$$u = \frac{k}{L} \frac{x_2 - x_1}{C}$$

(19)

Implementing the above control law (19), the simulation and experiments are plotted in the Fig. 12. The steady-state error is virtually eliminated but note the second-order type behavior with transient overshoot. This is to be expected. The boost converter is a second-order system, which becomes first order by virtue of the synergetic control law (5). Notice that the introduction of the integral term increases the system order by one, and returns the controlled system back to second-order.

The integral error term can also be added into the current-limited macro-variable (15). The macro-variable becomes

$$\psi = x_i + A \cdot \tanh \left[ \frac{-x_{1,ref} + (x_2 - x_{2,ref})/k}{A} \right] + k_2 \int (x_2 - x_{2,ref}) \, dt \quad (20)$$

Applying the usual synthesis procedure, the control law is obtained:

$$u = \frac{V_L}{L} + \frac{x_2}{kRC \cosh^2 y} + \frac{A}{T} \tanh y + k_1 (x_1 - x_{1,ref}) + k_2 \int (x_2 - x_{2,ref}) \, dt$$

$$u = \frac{x_2}{kC \cosh^2 y}$$

(21)

where $$y = \frac{-x_{1,ref} + (x_2 - x_{2,ref})/k}{A}$$

The simulation and experimental results for control law (21) are shown in Fig. 13. The addition of the integral term introduces a voltage overshoot. It is possible to avoid this by activating the integral error term only when the voltage error is below a certain threshold so that transient behavior is not affected.
D. Control Law using High Pass-filtered Inductor Current

An alternative control law can be defined that uses a high pass filtered measurement of the inductor current in place of the reference current \( x_{\text{ref}} \) (which, in most applications, is not actually known). The macro-variable becomes

\[
\psi = (x_2 - x_{\text{ref}}) + kx_1 s \frac{T}{1 + s T} = 0 \tag{22}
\]

Comparing the modified macro-variable (22) to the original macro-variable (5), the absence of the current reference value \( x_{\text{ref}} \) is apparent.

The high pass-filtered inductor current can also be combined with the current limit feature to limit the maximum value of the inductor current.

The simulation and experiments are shown in Fig. 14 with a high pass filter corner frequency of 100 Hz.

VI. CONCLUSION

The basic application of Synergetic theory to the control of a boost converter is introduced in [2]. In this paper variations of the basic control law were explored. Among these:

- Two different implementations of a control law that limits inductor current overshoot.
- A control law that includes dynamic adaptation of the control parameter depending on the output voltage error.
- A control law that includes an integral error term to eliminate steady-state error.
- A control law that uses the high pass-filtered inductor current and does not require knowledge of the reference current, which is load dependent for a desired output voltage.

These control features can also be combined together. Simulation and experimental results for all control laws are given. Good agreement between simulation and experimental results is found.

ACKNOWLEDGMENT

This work was supported by the US Office of Naval Research (ONR) under grant N00014-00-1-0131.

REFERENCES