Synergetic Control for DC-DC Buck Converters with Constant Power Load

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Abstract—An algorithm for design of synergetic control is derived, using as an example a system containing buck choppers with constant power load [1]. The closed loop system behavior is compared to that of the same system controlled by feedback linearization control. As a result, it is shown that synergetic control has better dynamics and a faster response. This system nullifies steady state error not only of output voltage but also in current sharing among parallel converters.

I. INTRODUCTION

The design of a new highly efficient autonomous power system, especially for applications in ships or aircrafts, is usually very expensive [2]. Some improved control system designs have been realized through DC zonal electrical distribution systems (DC ZEDS), which lower cost and weight, and improve mobility and survivability of the power system [2]. However, new requirements in system design have created new challenges for design of controller for DC power converters.

Using the classical approach for designing complex power systems, which is based on extensive modeling and expensive prototyping [3], one must overcome such obstacles as nonlinearity, multi-connectivity, and multidimensionality of the system. Some advanced methods such as the feedback linearization technique [1] or sliding mode control [4] allow designers to avoid some nonlinearity problems, but multiconnectivity and multidimensionality are still intractable issues.

Several software products are available for lowering the cost of prototyping. Among them is the Virtual Test Bed (VTB) software [5]. It offers a number of features to facilitate system design and testing, such as multi-formalism, co-simulation, experiment planning tools, high-level visualization, and support for hardware-in-the-loop. However, significant improvement of control design for nonlinear, multiconnected, and multidimensional systems is not possible without the use of modern control theory. The synergetic control theory used in this paper is one of the promising new options in modern control theory. It opens new horizons in DC power system design.

Synergetic control theory is based on ideas of modern mathematics and uses essential properties of nonlinear dynamic dissipative systems. This promising new approach allows analytical derivation of control laws that not only ensure stable operation of the closed loop system, but that also produce substantial order reduction by sequential decomposition of a high order, nonlinear, and multiconnected system [6,7,8]. As a result, the use of synergetic control theory improves system performance, a very important goal in design of high energy density power systems (PS).

In this paper, the algorithm of synergetic control design is presented, along with an example of its application to a system reported in [1] containing buck converters with constant power load. In [1] a control based on feedback linearization was applied to the system. In this paper we will compare the results of synergetic control with results of feedback linearization control reported in [1] by simulating both. The comparative modeling of the closed loop system with synergetic and feedback linearization controls is performed in VTB.

The paper is organized as follows. The Synergetic Control Design section presents an algorithm for design of synergetic controls. In the following section, synergetic control is applied to DC-to-DC converters with constant power load for two different cases: for one converter, and for two parallel-connected converters. The designs are based on averaged models of the converters. The Simulation Results section presents the results of comparative modeling for synergetic and feedback linearization controls.

II. SYNERGETIC CONTROL DESIGN

Synergetic control theory is a state space approach based on ideas of modern mathematics and synergetics. In short, utilizing dissipative structure algorithms [9], synergetic control theory supplies an analytical procedure for design of controls for nonlinear, multi-connected, high dimensional systems, which includes the essential properties of the controlled plant in the task statement. As a result, synergetic control theory allows designers to state and then efficiently solve many difficult control problems, which have been neither solved by other known methods nor even...
stated due to their complexity. These problems relate not only, for example, to global stability of the closed loop operation or global optimization of the system behavior but also to simplification of transition from one power sharing strategy to another, or minimization of power losses in the system.

The synergetic control design procedure is based on the method of analytical design of aggregated regulators – ADAR [6,7,8] – and is composed of a task statement and a control synthesis.

1) Task statement

For a dynamic system described by a system of nonlinear differential equations
\[ \dot{x} = f(x,u) \]  (1)
where \( x \) is the state vector of the system of size \( n \), \( f(.) \) is a continuous nonlinear function, and \( u \) is a control vector of size \( m \leq n \), we can define a control law in the following form
\[ u(\psi) = u(x) \],  (2)
which moves the representing point from arbitrary initial conditions \( x_0 \) in region \( \Gamma \in \mathbb{R}^n \) towards the specified invariant manifold
\[ \psi(x) = 0 \],  (3)
and along the manifold to the specified equilibrium point.

A very important part of the control design is defining the macro-variables \( \psi(x) \). Introducing an algebraic relation among the system variables, they reflect designer specifications. In a simple case they can be defined as a linear combination of the state variables. The number of macro-variables does not exceed the number of control channels.

2) Control Synthesis

The evolution of the macro-variable \( \psi(x) \) towards manifold (3) can be defined in various ways. The equation (4) defines this evolution and is used for control derivation in this paper.
\[ T \cdot \dot{\psi} + \psi = 0; \ T > 0 \],  (4)
where \( T \) is a vector of size \( m \) defining the speed of convergence of the macro-variables to the manifolds. Substituting system equations (1) into (4) gives:
\[ T \cdot \frac{d\psi}{dx} \cdot \dot{x} + x = 0 \].  (5)
Solving the system (5) for \( u(x) \) gives a control that ensures the specified properties and decomposes the system by contraction of its state space. The order of the system on the manifolds is \( n-m \).

The result of this synergetic control design procedure is an analytical control law that ensures stable motion to and along the introduced manifolds to the final or equilibrium point of the closed loop system. The trajectory of system motion is determined by the actual system trajectories on the manifolds, which depend on the structure of the manifold equations. The stability of the system in the final state can be checked by any appropriate method [10].

In the following section, an application of synergetic control design procedure is shown, using as an example a system containing two buck choppers connected in parallel to a constant power load.

III. SYNERGETIC CONTROL STRATEGIES FOR PARALLELED DC/DC CONVERTERS

Taking as a prototype the circuit shown in Fig. 1, which is the example used in [1], we next present a scalar control design for one converter and a vector control design for two converters. Each control design includes task statement, control synthesis, and stability analysis. We use a state space averaged model of the Buck converter, which is derived assuming:
- The buck converter operates in continuous conduction mode.
- Switching occurs at a very high frequency.
- The averaged capacitor voltage and inductor current are used as state variables from which switching harmonics are mathematically eliminated.
- Constant power load is mathematically represented by a current source with a value given by a constant input, \( P_{load} \), divided by the output voltage.
- Parasitic effects are ignored.

Fig. 1. Circuit of parallel connected buck converters with constant power load

A. Scalar control design suppressing piece-wise constant disturbance

1) Task statement

Consider the task of synthesizing a buck converter controller, which is to maintain specified output voltage \( V_{c,ref} \) under variation of operating conditions due to load or parameter changes. Change in operating conditions is accounted for by introduction into the system model of a function \( M(t) \). The state space averaged model of the converter working on a constant power load is presented in (6) where it is assumed that duty cycle consists of feedforward \( d_{ss} \) and feedback \( d_f \) components.
\[
\begin{align*}
\frac{dv_{c1}}{dt} &= \frac{1}{C_t} i_{c1} - \frac{v_{c1}}{R_{ext} \cdot C_t} - \frac{P_{load}}{v_{c1} \cdot C_t} + M(t) \\
\frac{di_{c1}}{dt} &= \frac{1}{L_t} v_{c1} + \frac{d_x E}{L_t} + \frac{d_y E}{L_t}
\end{align*}
\]  (6)
where: \( i_{c1} \) is inductor current, \( v_{c1} \) is voltage at the output of the converter, \( P_{load} \) is the load power. The capacitance \( C_t \) includes capacitance \( C_f \) plus the external capacitance \( C_{ext} \) of...
the connected loads (Fig. 1).

Usually, a power converter is subject to switching disturbances; in that case the function \( M(t) \) can be approximated by piecewise-constant disturbance the dynamic model of which is as follows:

\[
M(t) = k \cdot (v_{\text{ci}} - V_{\text{c.ref}})
\]

(7)

Following the same pattern as in [1], the local diffeomorphic state coordinate transformations (8) are introduced.

\[
M(t) = y(t)
\]

(8)

\[
z_1 = C_z (v_{\text{ci}} - V_{\text{c.ref}})
\]

\[
z_2 = \frac{C_z}{C_i} (i_{\text{Li}} - i_{\text{Li}0}) = \frac{C_z}{C_i} \left( i_{\text{Li}} - \frac{v_{\text{ci}}}{R_{\text{ref}}} - \frac{P_{\text{load}}}{v_{\text{ci}}} \right)
\]

These transformations pseudo-linearize the system and move the origin of the system into the operating point. The pseudo-linearization is not required for synergistic control design but it simplifies stability analysis of the closed loop system.

Thus, using the new variables (8), and extending the system by adding a model of the referenced piecewise-constant disturbance (7), the system (6) is rewritten in the following form [6]:

\[
y(t) = \eta z_1
\]

(9)

\[
z_1(t) = \frac{z_2}{z_4 + \gamma} + y
\]

\[
z_1(t) = \frac{z_2}{z_4 + \gamma} + \frac{C_z}{C_i} u_t + \frac{P_{\text{load}} C_z^2}{C_i} \left( \frac{z_4}{z_4 + C_z V_{\text{c.ref}}} \right)
\]

where \( u_t = \frac{E}{L_s} \cdot d_c \).

The essence of the extended model (9) is that the nonlinearity of the system can be compensated by control \( u_t \); the equilibrium state of the system is at the origin of coordinates; due to the introduced additional state variable, the system is capable of rejecting piece-wise constant disturbances and obtaining zero steady state error. Thus, system (9) incorporates all requirements of the control design.

2) Control synthesis

To synthesize the control, the following macro-variable is introduced:

\[
\psi_1(x) = \alpha \cdot z_1 + z_2 + \gamma \cdot y
\]

(10)

Assume that the desired dynamics of evolution of the macro-variable \( \psi_1(x) \) is:

\[
\psi_1(x) = 0
\]

(11)

\[
T_1 \cdot \psi_t + \psi_1 = 0
\]

(12)

The control law is calculated by substituting the expression for macro-variable (10) into the functional equation (12), accounting for the system model (9), and solving the resulting system jointly. The following control law results.

\[
\begin{align*}
\frac{C_z}{C_i} u_t &= \frac{z_2}{L_s C_z} + \frac{1}{R_{\text{ref}} C_i} \cdot z_3 + \\
&= \frac{P_{\text{load}} C_z^2}{C_i} \cdot \left( \frac{z_4}{z_4 + C_z V_{\text{c.ref}}} \right) - \alpha \cdot \gamma \cdot \eta \cdot z_1 - \frac{1}{T_1} \psi_t
\end{align*}
\]

(13)

3) Stability of the Closed Loop System

The control law (13) moves the plant (9) from the arbitrary initial conditions to the manifold (11). Motion along this manifold is described by the reduced order dynamic system (14) where here and later in the paper, subscript \( \psi \) indicates that the representing point of the plant is on the manifold.

\[
y_\psi(t) = \eta z_\psi
\]

(14)

\[
z_\psi(t) = z_{\psi\psi} + y_\psi
\]

Substitution of the relation \( z_{\psi\psi} = \alpha z_{\psi\psi} - \gamma y_\psi \) from (11) into (14) yields

\[
y_\psi(t) = \eta z_\psi
\]

(15)

Writing the system (15) as a single equation results in:

\[
z_\psi(t) + \alpha z_\psi(t) + (1 - \gamma) y_\psi = 0.
\]

(16)

From the equation (16) it follows that motion along manifold (11) is asymptotically stable for:

\[
\alpha > 0; \quad (1 - \gamma) \eta > 0
\]

(17)

Thus, if the inequalities (17) and \( T_1 > 0 \) are satisfied, the synthesized closed-loop system will also possess the property of asymptotic stability.

To illustrate the properties of the system motion along the manifold, equation (16) is rewritten in the following form:

\[
z_\psi(t) + \alpha z_\psi(t) + (1 - \gamma) \eta \int z_\psi dt = 0.
\]

(18)

Equation (18) shows that on the manifold (11) there is a PI control law for the coordinate \( z_{\psi\psi} \). An analogous PI law is formed for the coordinate \( z_{\psi\psi} \). Thus, according to (17) and (18), the synthesized control law \( u_t \), (13), besides giving the semi-global asymptotic stability, also suppresses the piece-wise constant disturbance \( M(t) = \text{const} \) that influences the converter.

To account for the limitation \( 0 < d_c + d_s \leq 1 \), the control law (13) can be formulated in the following expressions:

\[
u_{\text{sup}} = 0.5 \cdot (\tanh(\eta \psi_t) + |\tanh(\eta \psi_t)|)
\]

or

\[
u_{\text{sup}} = 0.5 \cdot 1 - e^{|\psi_t|} + |1 - e^{-|\psi_t|}|
\]

(19)

Thus, application of the synergetic control theory [7] allows synthesis of control laws ensuring asymptotic stability and suppressing piece-wise constant disturbance for the buck converter with constant power load.

B. Vector control design for two paralleled converters

1) Task statement

The control synthesis task for two buck converters is similar to that for one-converter system, however, it requires to maintain not only the specified output voltage \( V_{\text{c.ref}} \) but...
also the equal current sharing \( i_{L1} = i_{L2} \), while the system is affected by piece-wise constant disturbance \( M(t) \). The state space averaged model describing the shunt connection of buck converters is shown in (19) where controls are represented by feedforward \( d_{ct} \) and feedback \( d_{c2}, d_{c1} \) components of duty cycle.

\[
M(t) = k \left( v_{ci} - V_{ref} \right)
\]

\[
\frac{dv_{ci}}{dt} = \frac{1}{C_i} \left( i_{L1} + i_{L2} \right) - \frac{v_{ci}}{R_{cct}} - \frac{P_{out}}{v_{ci}} + M(t)
\]

\[
\frac{di_{L1}}{dt} = \frac{1}{L_1} v_{ci} + \frac{d_c E}{L_1} + \frac{d_c E}{L_1}
\]

\[
\frac{di_{L2}}{dt} = \frac{1}{L_2} v_{ci} + \frac{d_c E}{L_2} + \frac{d_c E}{L_2}
\]

(19)

where \( i_{L1}, i_{L2} \) are currents in inductors \( L1 \) and \( L2 \) respectively, \( v_{ci} \) is the voltage at the output of the converters, \( P_{out} \) is the load power. The variable capacitance \( C_i \) includes capacitance \( C_i \) and \( C_p \) plus the external capacitance \( C_{ext} \) of the connected loads.

To the system (19) including the model of the piece-wise constant disturbance (7), the coordinate transformations (20) are used to move the origin of the system into the equilibrium point [1] where the specified output voltage \( z_1 = 0 \) and current sharing \( z_3 = 0 \) are maintained.

\[
M(t) = y_1(t)
\]

\[
z_1 = (C_i + C_p) \left( v_{ci} - V_{ref} \right)
\]

\[
z_2 = \frac{C_i + C_p}{C_i} \left( i_{L1} + i_{L2} - \frac{v_{ci}}{R_{cct}} \right)
\]

\[
z_3 = i_{L1} - i_{L2}
\]

(20)

Applying these transformations to the system (19) yields as follows:

\[
y_1(t) = \eta_2 z_1
\]

\[
y_2(t) = z_2 + y_3
\]

\[
y_3(t) = -a_1 z_1 + a_2 z_1 + a_3 z_1 - \left( C_i + C_p \right) \left( v_{ci} - V_{ref} \right)
\]

\[
y_4(t) = h_3 z_3 + u_3
\]

(21)

where:

\[
a_1 = \frac{1}{C_i} \left( \frac{1}{L_1} + \frac{1}{L_2} \right)
\]

\[
a_2 = \frac{1}{R_{cct}} C_i
\]

\[
a_3 = \frac{C_i + C_p}{C_i} \left( P_{out} \right)
\]

\[
h_3 = \frac{1}{C_i + C_p} \left( \frac{1}{L_2} - \frac{1}{L_1} \right)
\]

\[
u_3 = E \left( \frac{d_{c1}}{L_1} \frac{d_{c2}}{L_2} \right)
\]

The system (21) accounts for control design specifications such as current sharing and output voltage regulation, and also incorporates the disturbance model (7).

2) Vector control synthesis

In order to synthesize the controller for the two-converter system, macro-variables (22) and (23) are introduced.

\[
\psi_2 = \alpha_2 z_1 + z_2 \quad \alpha_2 + \alpha_4 y_1
\]

(22)

\[
\psi_2 = \beta_2 z_1 \quad \beta_2 + \beta_4 y_1
\]

(23)

Then, if \( T_2 > 0 \), \( T_3 > 0 \) are satisfied, the synthesized vector control laws (22) and (23) towards corresponding manifolds (25).

\[
\psi_2 = 0 \quad \psi_3 = 0
\]

(25)

Vector control laws are derived by substituting the macro-variables (22) and (23) into functional equations (24) and solving them jointly, accounting for the system model (21). The resulting controls are as follows:

\[
\lambda u_2 = \frac{1}{a_2 - \beta_2} \left( (a_2 \beta_2 - a_2 \beta_1) y_2 - \frac{z_1}{T_1} \right) + \left( (a_2 \beta_2 - a_2 \beta_1) (y_2 + y_3) - \frac{a_2 \beta_1}{T_1} \right) \psi_2
\]

\[
+ \left( a_2 z_1 + a_2 z_2 - \frac{a_2 \beta_2}{[z_1 + (C_i + C_p) \psi_2 + v_1]} \right)
\]

\[
u_1 = -h_3 z_3 + \frac{1}{a_3 - b_3} \left( (\beta_3 - a_3) y_3 - \frac{z_3}{T_3} \right) \psi_3
\]

(26)

where \( y_1 = \eta_2 z_1 \), \( y_2 = z_2 \) and feedback control laws \( u_2 (26) \) and \( u_3 (27) \) move the system representing point onto the seam of the manifolds (25). Motion of the system (19), (26) and (27) along this manifold is described by the following reduced order dynamic system:

\[
y_2(t) = \eta_2 z_2
\]

(28)

Substituting into (28) the coordinate \( z_2 \), the joint solution of the equations (25), yields

\[
y_2(t) = \eta_2 z_2
\]

(29)

\[
z_2 = \frac{a_2 \beta_1 - a_2 \beta_3}{a_2 - \beta_3} y_2 + \left( \frac{a_2 \beta_1 - a_2 \beta_3}{a_2 - \beta_3} + 1 \right) y_2
\]

(30)


The stability conditions for these equations are as follows:

\[
\frac{a_2 \beta_1 - a_2 \beta_3}{a_2 - \beta_3} > 0 \left( \frac{a_2 \beta_1 - a_2 \beta_3}{a_2 - \beta_3} + 1 \right) y_2 > 0
\]

(30)

Thus, if the inequalities (30) and \( T_2 > 0 \), \( T_3 > 0 \) are satisfied, the synthesized vector control laws \( u_2 (26) \) and \( u_3 (27) \) will ensure semi-global asymptotic stability of the closed-loop system (19), (26), (27) with respect to the state \( z_{i1} = z_{i2} = z_{i3} = 0 \).
In addition, these laws suppress the uncontrolled piecewise-constant disturbance $M(t) = \text{const}$, which is illustrated by restating (29) in the following form:

$$
\dot{z}_{up}(t) = \frac{a_1 \beta_1 - a_2 \beta_2}{a_5 - \beta_5} z_{up} + \left( \frac{a_1 \beta_1 - a_2 \beta_2}{a_5 - \beta_5} \right) \eta \int z_{up} dt \tag{31}
$$

Thus, the vector controller synthesized using synergetic control theory has two control channels $u_2$ (26) and $u_3$ (27), and it ensures asymptotic stability of the two buck converters connected to a common constant power load and suppresses the piece-wise constant disturbance. Note that according to (24) and (29), the duration and properties of transients in the synthesized system (19), (26), (27) are determined by the parameters $T_2$, $T_3$, $\eta_1$, $\alpha$, $\beta_1$ selected at the control design time. These parameters can be calculated using, for example, indexes such as ISE or ITAE [10].

IV. SIMULATION RESULTS

Comparative simulation of synergetic and feedback linearization controls has been performed in VTB software. This public domain software was chosen based on the following features: a) extensive model coverage; b) ability to use different languages for model description (Matlab/Simulink, ACSL); c) experiment planning tools; d) hardware-in-the-simulation-loop support. All these features, combined with rapid prototyping technology [11], facilitate the system test and design.

The coefficients of the control laws were calculated based on system requirements and using indexes from [12] and were checked by simulation. A series of simulations experiments was performed for various load conditions representing extreme cases of the load ranges, which are given in the system definition for constant power load, resistive, and capacitive components of the load. In simulation results, the envelopes of the curves obtained for different load values are shown, allowing a comparison for the whole set of the load conditions.

B. One buck converter

To compare the synergetic controls to the feedback linearization control, the same circuit as in [1] is used. A buck converter network has the following parameter values: $L_1 = 1.35$ mH, $C_1 = 2.600$ µF, $V_{\text{ref}} = 750$ V, and an input voltage of $E=850$V. The load is uncertain with parameters falling in the following bounds:

- $5.625 \Omega \leq R_{\text{ext}} \leq \infty$;
- $2610 \mu F \leq C \leq 8200$ µF;
- $0$ kW $\leq P_{\text{load}} \leq 100$ kW.

To achieve a robust design, it is important to choose control law coefficients $\alpha_1$, $\gamma$, $T_i$, $\eta_1$ properly. The values used are: $\alpha_1 = 5000$, $\gamma = 3.0$, $T_1 = 0.0067$, $\eta_1 = 50$. In order for the scalar synergetic control law to be used in controller design, inverse mapping (13) is necessary. The resulting controls are as follows:

$$
d_{sc} = -0.0019 \cdot (v_{c1} - V_{cl,\text{ref}}) - 0.00409 \cdot (i_{c1} - i_{\alpha}) + 0.0001074 \cdot \frac{i_{c1} - i_{\alpha}}{v_{c1}} - 0.0086 \cdot \gamma
$$

$$
d_{FL} = -0.00326 \cdot (v_{c1} - V_{cl,\text{ref}}) - 0.005 \cdot (i_{c1} - i_{\alpha}) - 23.2 \cdot \frac{i_{c1} - i_{\alpha}}{v_{c1}} \tag{33}
$$

where $d_{sc}$ stands for synergetic control and $d_{FL}$ stands for feedback linearization control.

The family of curves obtained from the simulation is presented in Fig. 2. The combinations of load resistance and capacitance are as follows: $R_{\text{ext}} = 5.625 \Omega$, $C_1 = 5.210$ µF; $R_{\text{ext}} = 50 \Omega$, $C_1 = 5,110$ µF; $R_{\text{ext}} = 5.625 \Omega$, $C_1 = 10,800$ µF. The system is initially energized with $P_{\text{load}} = 0$ kW and allowed to operate in the steady state up to 0.09 s. At 0.09 s $P_{\text{load}}$ is stepped to 75 kW. At 0.12 s $P_{\text{load}}$ is stepped to 100 kW. At 0.14 s $P_{\text{load}}$ is stepped down to 0 kW.

Thus, the simulation experiments show that the scalar synergetic control law ensures stable and robust operation for the closed loop system, and nullifies steady state error of the output voltage. Moreover, it has faster output voltage response at turn-on than feedback linearization control, and is less sensitive to load variation.
C. Two paralleled buck converters

A buck converter network with the same circuit as in [1] has the following parameter values: \( L_1 = 1.35 \text{ mH} \), \( L_2 = 1.25 \text{ mH} \), \( C_1 = 2.600 \text{ µF} \), \( C_2 = 2.500 \text{ µF} \), \( V_{in}\text{ref} = 750 \text{ V} \), and an input voltage of \( E = 850 \text{ V} \). The load is uncertain with parameters falling in the following bounds:

\[
2.81 \Omega \leq R_{\text{ext}} \leq 50 \Omega;
\]

\[
5,110 \mu\text{F} \leq \ C_{\text{ext}} \leq 10,000 \mu\text{F};
\]

\[
0 \text{ kW} \leq P_{\text{load}} \leq 200 \text{ kW}.
\]

It is desired that the output voltage be regulated at 750 V, that the current be equally divided between the two units, and that the transient response be such that the settling time is less than 10 msec and the overshoot is less than 6 %.

In order to obtain these characteristics, control law coefficients are chosen to be the following: \( \alpha_1 = 5500 \), \( \beta_1 = 5500 \), \( \alpha_2 = 2.0 \), \( \beta_2 = 1.0 \), \( \alpha_3 = 3.0 \), \( \beta_3 = 3.0 \), \( T_{\text{s}} = 0.003 \), \( \eta_2 = 45 \).

As a result, inversely mapped control laws for parallel connected buck converters are of the following form:

\[
d_{\text{SC}} = -A \cdot (v_2 - V_{\text{in}\text{ref}}) - B \cdot (i_{\text{L1}} - i_{\text{L2}}) +
\]

\[
-D \cdot i_{\text{yf}} - E \cdot \frac{V_{\text{L}}}{V_i} - F \cdot y
\]

(34)

where \( i_{\text{yf}} = (i_1 + i_2) - (i_{\text{L1}} + i_{\text{L2}}) \)

Control laws coefficients are shown in Table 1.

<table>
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<th>Coeff.</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \alpha_3 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
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</tr>
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<td>34.26</td>
<td>31.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.0035</td>
<td>0.003244</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next, the designed systems have been simulated using an averaged representation of the buck converters. The series resistance of each inductor is 0.5 mΩ. For each control, four simulations for extreme resistance and capacitance values have been performed. The combinations of load resistance and capacitance are as follows: \( R_{\text{ext}} = 2.81 \Omega \), \( C_{\text{ext}} = 5,110 \mu\text{F} \), \( R_{\text{ext}} = 50 \Omega \), \( C_{\text{ext}} = 5,110 \mu\text{F} \), \( R_{\text{ext}} = 2.81 \Omega \), \( C_{\text{ext}} = 10,000 \mu\text{F} \), \( R_{\text{ext}} = 50 \Omega \), \( C_{\text{ext}} = 10,000 \mu\text{F} \). The system is initially energized with \( P_{\text{load}} = 0 \text{ kW} \) and allowed to operate in the steady state up to 0.09 s. At 0.09 s \( P_{\text{load}} \) is stepped up to 150 kW. At 0.12 s \( P_{\text{load}} \) is stepped up to 200 kW. At 0.15 s \( P_{\text{load}} \) is stepped down to 0 kW.

It can be seen from Fig. 3 that the closed loop system with synergetic control shows solid, stable, and fast response and at the same time better accuracy of output voltage than the system with feedback linearization control. The solid, stable and fast response is based on decomposition of the closed loop system. In the feedback linearization case the closed system is of the fourth order. The closed loop system with synergetic control law starts from the fourth order and through influence of the control it becomes a second order system. As a result, the closed loop system with synergetic control does not have significant delay during switching on and has better transient response. Integral action introduced in the synergetic control law allows the system to suppress load uncertainty and as a result, improve accuracy of the output voltage. Because of the small value of \( \eta_2 \), the integration is slow.

Table 1.

![Switch on transients of the output voltage when constant power load steps up form 0 % to 75 % of maximum value](image)

![Switch off transients of the output voltage when constant power load steps down from 100 % to 0 %](image)

An important characteristic of the controls is current sharing capabilities. In Fig. 4 current sharing is presented for corresponding switching conditions.

![Current sharing in the closed loop system](image)

It can be seen in Fig. 4 that the steady state error of current sharing becomes 0.1 % (at 0.125 s) within 0.0115 s after full load connection at 0.11 s. The modeling shows that in opposite to feedback linearization control case when the current sharing error is proportional to the load current, this...
error in the system with synergetic control decays with time. The final value of the error is limited by the accuracy of implementation of the integral action and by error introduced by sensors. Ideally, the integral action introduced in the control law allows the closed loop system to nullify the steady state error and as a result, improve reliability of the system.

D. Discontinuous conduction mode operation

The assumption that buck converters operate in continuous conduction mode is very restricting and limits the scope of control application. In reality, buck converters often work in discontinuous conduction mode (DCM). Switching between operating modes may cause system to collapse [13]. Because of this we explored the capability of synergetic control laws to minimize the effects of discontinuities in buck converter operation. During simulation a state space averaged model from [14] available in VTB accounted for discontinuous conduction operation in the buck converter. Fig. 5 shows the steady state voltage of one-converter system in DCM.

Fig. 5. Output voltage of one-converter system in DCM regime operation of the buck converter when \( T = 0.0002 \) s

The output steady state voltage in DCM has a periodic form. The oscillation of the output voltage corresponds to the inadequacy of the models to predict system behavior in DCM operation. However, by incorporating integral action into synergetic control law the closed loop system can maintain output voltage within specified limits. Moreover, the magnitude of output voltage oscillations can be controlled by the value of time constant \( T \). By decreasing this parameter the specified accuracy of output voltage and current sharing can be achieved.

V. CONCLUSION

We have presented a nonlinear synergetic control algorithm for an isolated buck chopper and two parallel buck choppers supplying a constant power load. The closed loop system shows stable operation, fast response, good voltage regulation, and the ability to nullify steady state error not only of output voltage but also of current sharing. The synergetic control shows better performance than feedback linearization control [1] on the particular test system.

Synergetic control theory provides a very efficient method for designing controls for complex power systems such as DC ZEDS. The synergetic approach allows designers to overcome the problems of complex systems such as nonlinearity, multiconnectivity, and high dimensionality. All these synergetic control theory capabilities, combined with VTB rapid prototyping technology [11], facilitate the system test and design.

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REFERENCES