Propagation of uncertainty through signal flow simulation using Polynomial Chaos Theory

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Abstract

Modeling the effects of uncertainty is important in many fields. Thus, there is a drive to develop tools to analyze and simulate the effects of uncertainty on both small and large scale system. Polynomial Chaos Theory (PCT) represents an interesting way to handle to inclusion of uncertainty in the modeling and simulation process.

PCT allows for the development of a probabilistic model of the system in a deterministic setting. This is done by using random variables and orthogonal polynomials to handle the effects of uncertainty.

The goal of this paper is to introduce a formal method to build an uncertain simulation model of system starting from a library of component models. In particular we focus on simulation based on signal flow coupling. The components are then defined in terms of state equations and output function.

The extension here proposed presents an original definition of the output function for which the model outputs are actually the co-efficient of the PCT expansion, in the following referred as uncertain states. The uncertain states of one model block are then propagated to a connected block. This approach allows for model reusability and definition of more complex scenarios starting from simple component models.

1. INTRODUCTION

Polynomial Chaos Theory (PCT) allows for parametric uncertainty to be formally included in the modeling process [1]. PCT is a spectral expansion of random variables that approximates a random process by means of a complete and orthogonal polynomial basis of random variables [2][3][4]. This spectral expansion contains all the possible outcomes in the form of a polynomial representation. Applications of PCT have been discussed in fields such as fluid dynamics [3], measurement uncertainty [1][6][8][9][14], ,power electronics and circuit simulation [6], entropy multivariate analysis [9], control design [10][11][12], design of a Two-Planar Manipulator [13] and polynomial chaos based observers for use in control theory [14]. In [15] an overview of applications of PCT to the electrical engineering domain is discussed.

As reported in literature, the size and complexity of the PCT formulation can grow quite significantly affecting the computation effort [8]. This problem is particularly important if we use circuit-based modeling and then a matrix inversion process to perform dynamic simulation. For this reason, in this paper we explore how to build system models starting from component models based on signal coupling paradigm. The signal coupling paradigm is less affected than natural coupling by the growth of complexity. On the other hand, in comparison with other methods proposed for natural coupling, we need to provide a methodology that preserves the possibility to build component models independently, propagating the uncertainty at system level only at run-time.

In signal flow simulations each component/model calculates its own output(s) based on its input(s) and propagate the output to next block. This method of simulation simplifies the overall complexity of the simulation. This paper aims at demonstrating that through the use of signal flow it may be possible to propagate uncertainty from one signal flow block to another while preserving the overall uncertainty output of the entire system. The method of propagating the effects of uncertainty would reduce complexity and size of the PCT expansion, since one would be looking at system block by block. Given this possibility, it would be than possible to do worst-case analysis, reliability, quality assurance testing, etc. without resorting to such tools as Monte Carlo simulation in many practical cases. It should be clarified though that, in any case, when the number of uncertainty grows over a certain limit it could still be convenient to move to collocation method and even Monte Carlo analysis.

The paper is structured as follows: section 2 provides a brief introduction about PCT while section 3 describes how to expand a system using PCT. Section 4 describes cascading the PCT models and section 5 describes closing the loop with PCT models. Section 6 is the conclusion.

2. POLYNOMIAL CHAOS THEORY

In 1938 by N. Wiener developed a concept that uses a spectral expansion of random variables that approximates the random process using Hermite polynomials. He termed his idea homogeneous chaos [16]. With homogenous chaos Wiener used the Hermite orthogonal polynomials in a stochastic space to represent and propagate uncertainty of a Gaussian process [16]. Wiener's idea was later expanded to include the whole Askey-scheme of orthogonal polynomials and was renamed Wiener-Askey polynomial chaos [17]. Using PCT, the spectral expansion of a random process can be described as:

$$g(\xi_1,..,\xi_i) = \sum_{n=0}^{\infty} x_n \Phi_n(\xi_1,..,\xi_i) \tag{2.1}$$

where: $g(\xi)$ is the random process or function under analysis, x_n are the coefficients of the expansion, Φ_n are the polynomials of the selected base, and ξ_i are random variables with a PDF defined according to the polynomial base (a convenient combination-PDF base minimizes the computational effort). This spectral expansion is an infinite series but for practical purposes must be limited to a finite number P of terms.

$$g(\xi_1,..,\xi_i) = \sum_{n=0}^{P} x_n \Phi_n(\xi_1,..,\xi_i) \tag{2.2}$$

The value of P of the truncated PCT expansion is determined by two factors: 1) the number of independent sources of uncertainty (n_v) and 2) the maximum order for the polynomial base (n_p) (the total number of terms). The value of P of the truncated PCT expansion is given as:

$$P = \left(\frac{(n_v + n_p)!}{n_v! n_p!} \right) - 1 \tag{2.3}$$

3. PCT EXPANSION

The PCT expansion of a system involves a sequence of steps. They can be summarized as follows:

- Choosing the appropriate basis and PCT order
- Expanding the uncertainty variable(s)
- Substituting uncertainty variable(s) in the governing equation
- Using the Galerkin projection method to find the coefficients of the PCT expansion.

This process determines the creation of an expanded deterministic model that can be analyzed and executed by using traditional simulation methods. When the number of uncertainties grows significantly the Galerkin projection is usually substituted with the collocation approach [18][19]. A comprehensive comparison between Galerkin approach and collocation method can be found in [20].

Using the state-space approach the order of the PCT expanded system is higher than the original system due to inclusion of the uncertain states. The order of the PCT expanded system is given by:

$$order = N \times P \tag{3.1}$$

where P is given in equation (2.3) and N is the order of the original system.

4. CASCADING PCT UNCERTAINTY MODELS

Consider a cascaded state-space model as in Figure 1 where in the state matrix (***A***,***F***) and/or the input matrix (***B***,***G***) matrix may contain uncertain parameters. One method to obtain the PCT uncertainty model is to derive the single model from Figure 1 as in Figure 2 and then expand the system using PCT (Figure 3). This paper proposes a method to expand System 1 using PCT and propagate its uncertain states to a PCT expanded model of System 2, as seen in Figure 4.

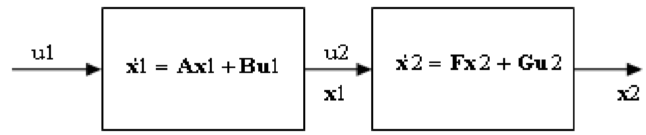


Figure 1. Shows a block diagram of a generic cascade state-space model.

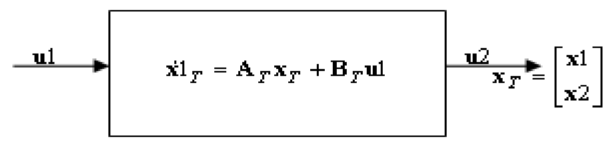


Figure 2. Shows a block diagram of a single model description of figure 1.

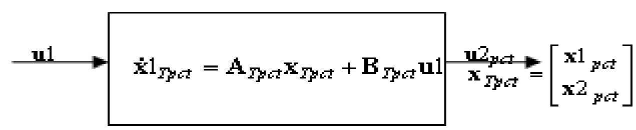


Figure 3. Shows a block diagram of a PCT expansion of figure 2.

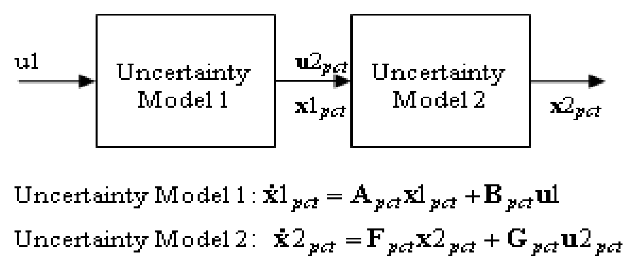


Figure 4. Shows the block diagram of a cascade PCT model of figure 3.

4.1. Methodology

Usually system models can be described as a proper combination of component models. In a signal flow representation, the main mechanism of signal propagation is the cascade connection.. In each block of these cascaded systems one or more uncertain parameters may be present.

Consider a system, S1, with one or more uncertain parameters, series cascaded with another system S2, which may also have one or more uncertain parameters.

As result of the cascade connection the following uncertainty propagation occurs: Let us consider the generic situation reported in the figure:

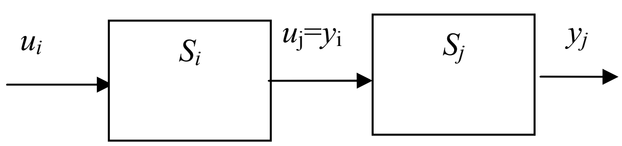


Figure 5: Generic cascade connection

The input u_i may be uncertain for the influence of other system connected as input to S_i. The expression of u_i in the PCT domain can be directly be inserted in the state space equation of S_i. As result of that the state of S_i will be expanded. Furthermore, S_i itself may have internal uncertainties expanding the dimension of the base even more. The calculation of the expanded S_i model can be easily calculated with automatic processes [21].

As a consequence of the expanded state definition for S_i the corresponding output y_i will be affected by the new extended state definition. But y_i=u_j, and then a similar process can be repeated for the model S_j.

The advantage of this process is that each system block can be considered independently, substituting the uncertain variables into the governing equation and performing the Galerkin projection to obtain the PCT expansion of the system.

4.1.1. Example: Cascading two first order systems.

Consider the system G_T(s) which is the series cascade of

$$G_1(s) = \frac{1}{s+a} \qquad G_2(s) = \frac{1}{s+b} \tag{4.1}$$

As seen in Figure 6

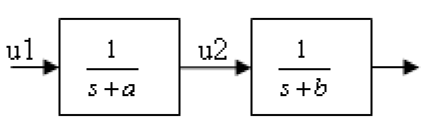


Figure 6: a simple cascade example

Equation (4.1)a can be written in state-space as

$$\dot{x}1 = ax1 + u1 = \boldsymbol{A}x1 + \boldsymbol{B}u1 \tag{4.2}$$

And Equation (4.1)b can be written in state-space as

$$\dot{x}2 = bx2 + u2 = \boldsymbol{F}x2 + \boldsymbol{G}u2 \tag{4.3}$$

As a transfer function the cascade system is given as :

$$G_T(s) = \frac{1}{(s+a)(s+b)} \tag{4.4}$$

The state-space formulation can be given as

$$\begin{aligned} \dot{x}1 &= ax1 + u1 \\ \dot{x}2 &= bx1 + x1 \end{aligned} \tag{4.5}$$

Considering equation (4.2) and assuming that the uncertain variable is *a,* which is uniformly distributed, the PCT expansion of this variable is given as:

$$a = \sum_{k=0}^{P} a_k \Phi_k(\zeta_1) \tag{4.6}$$

where Φ is the choice of basis (Legendre in this case) and ζ_1 is the assigned random variable for the uncertain parameter *a*.

Since we are cascading the first with another system with one uncertain variable, the PCT expansion of the dependant variable $x1$ is given as:

$$x1 = \sum_{k=0}^{P} x1_k \Phi_k(\zeta_1, \zeta_2) \tag{4.7}$$

where ζ_2 is the assigned random variable for the uncertain parameter *b*.

Substituting equation (4.6) and (4.7) into equation (4.2)

$$\dot{x}1 = \sum_{j=0}^{P} a_j \Phi_j(\zeta_1) \sum_{k=1}^{P} x1_k \Phi_k(\zeta_1, \zeta_2) + u1 \tag{4.8}$$

Taking the Galerkin projection

$$\dot{x}1_n = \frac{\int_{-1}^{1}\int_{-1}^{1} \dot{x}1(\zeta_1, \zeta_2) w \Phi_n d\zeta_1 d\zeta_2}{\int_{-1}^{1}\int_{-1}^{1} \Phi_n^2 d\zeta_1 d\zeta_2} \tag{4.9}$$

The first order expansion of this system is given as

$$\begin{bmatrix} \dot{x}1_0 \\ \dot{x}1_1 \end{bmatrix} = \begin{bmatrix} a_0 & 0 \\ 0 & a_0 \end{bmatrix} \begin{bmatrix} x1_0 \\ x1_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u1 = \boldsymbol{A}_{pct} \boldsymbol{x}1_{pct} + \boldsymbol{B}_{pct} u1 \tag{4.10}$$

Considering equation (4.3) and assuming that the uncertain variable is *b,* which is uniformly distributed, the PCT expansion of this variable is given as:

$$b = \sum_{k=0}^{P} b_k \Phi_k(\zeta_2) \tag{4.11}$$

Since we are cascading the first with another system with one uncertain variable, the PCT expansion of the dependant variable $x2$ is given as:

$$x2 = \sum_{k=0}^{P} x2_k \Phi_k(\zeta_1, \zeta_2) \tag{4.12}$$

The inputs of this system are the uncertain states of the previous system. Therefore

$$u2 = x1 = \sum_{k=0}^{P} x1_k \Phi_k(\zeta_1, \zeta_2) \tag{4.13}$$

Substituting equation (4.11), (4.12) and (4.13) into equation (4.3)

$$\begin{aligned} \dot{x}2 &= \sum_{j=0}^{P} b_j \Phi_j(\zeta_2) \sum_{k=1}^{P} x2_k \Phi_k(\zeta_1, \zeta_2) \\ &+ \sum_{l=1}^{P} x1_l \Phi_l(\zeta_1, \zeta_2) \end{aligned} \tag{4.14}$$

Taking the Galerkin projection

$$\dot{x}2_n = \frac{\int_{-1}^{1}\int_{-1}^{1} \dot{x}2(\zeta_1, \zeta_2) w \Phi_n d\zeta_1 d\zeta_2}{\int_{-1}^{1}\int_{-1}^{1} \Phi_n^2 d\zeta_1 d\zeta_2} \tag{4.15}$$

The first order expansion of this system is given as

$$\begin{aligned} \begin{bmatrix} \dot{x}2_0 \\ \dot{x}2_1 \end{bmatrix} &= \begin{bmatrix} b_0 & \frac{1}{3} b_1 \\ b_1 & b_0 \end{bmatrix} \begin{bmatrix} x2_0 \\ x2_1 \end{bmatrix} + \begin{bmatrix} x1_0 \\ x1_2 \end{bmatrix} \\ &= \mathbf{F}_{pct} \mathbf{x2}_{pct} + \mathbf{G}_{pct} \begin{bmatrix} x1_0 & 0 \\ 0 & x1_1 \end{bmatrix} \\ y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x2_0 \\ x2_1 \end{bmatrix} = H_{pct} \begin{bmatrix} x2_0 \\ x2_1 \end{bmatrix} \end{aligned} \tag{4.16}$$

To analytically check if our solution is correct let's cascade the two PCT system (equation (4.10) and (4.16)).

$$\begin{aligned} \dot{\mathbf{x}}_{Tpct} &= \begin{bmatrix} \mathbf{A}_{pct} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{pct} \end{bmatrix} \begin{bmatrix} \mathbf{x1}_{pct} \\ \mathbf{x2}_{pct} \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{B}_{pct} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{pct}\mathbf{C}_{pct} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{x1}_{pct} \end{bmatrix} \end{aligned} \tag{4.17}$$

which can be simplified as:

$$\dot{\mathbf{x}}_{Tpct} = \begin{bmatrix} \mathbf{A}_{pct} & \mathbf{0} \\ \mathbf{G}_{pct}\mathbf{C}_{pct} & \mathbf{F}_{pct} \end{bmatrix} \begin{bmatrix} \mathbf{x1}_{pct} \\ \mathbf{x2}_{pct} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{pct} \\ \mathbf{0} \end{bmatrix} u \tag{4.18}$$

It follows that:

$$\begin{bmatrix} \dot{x}1_0 \\ \dot{x}1_2 \\ \dot{x}2_0 \\ \dot{x}2_1 \end{bmatrix} = \begin{bmatrix} a_0 & 0 & 0 & 0 \\ 0 & a_0 & 0 & 0 \\ 1 & 0 & b_0 & \frac{1}{3} b_1 \\ 0 & 1 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} x1_0 \\ x1_1 \\ x2_0 \\ x2_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u1 \tag{4.19}$$

It can be shown that the PCT expansion of equation (4.5) is the same as equation (4.19). It can also be noted that equation (4.9) and (4.15) are identical to those used for the PCT expansion of equation (4.5).

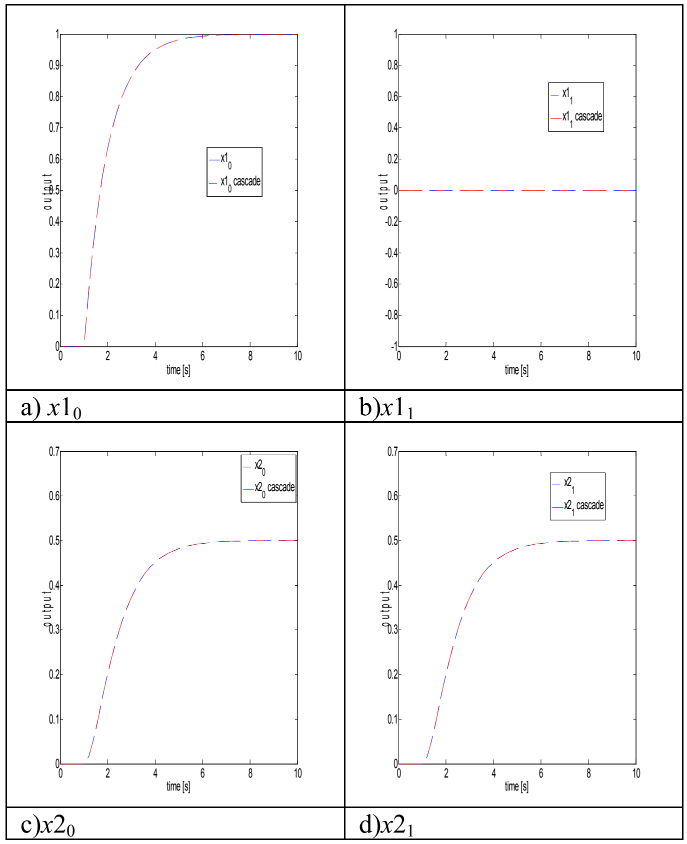


Figure 7: Comparison of a first order expansion PCT outputs for the cascaded blocks. Given a_0=-1, a_1=0.1, b_0=-2, b1=0.1

5. PCT UNCERTAINTY MODELS IN A FEEDBACK LOOP

Consider a generic system with no direct feed through i.e. input-output coupling matrix,**D**, is **0**:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \tag{5.1}$$

with a controller

$$\begin{aligned}\dot{\mathbf{x}}_c &= \mathbf{A}_c\mathbf{x}_c + \mathbf{B}_c\mathbf{u}_c \\ \mathbf{y}_c &= \mathbf{C}_c\mathbf{x}_c\end{aligned} \qquad (5.2)$$

The closed-loop system can be described as

$$\begin{aligned}\begin{bmatrix}\dot{\mathbf{x}} \\ \dot{\mathbf{x}}_c\end{bmatrix} &= \begin{bmatrix}\mathbf{A} & -\mathbf{BC}_c \\ \mathbf{B}_c\mathbf{C} & \mathbf{A_c}\end{bmatrix}\begin{bmatrix}\mathbf{x} \\ \mathbf{x}_c\end{bmatrix} + \begin{bmatrix}\mathbf{B} \\ \mathbf{0}\end{bmatrix} ref \\ \mathbf{y} &= \begin{bmatrix}\mathbf{C} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x} \\ \mathbf{x}_c\end{bmatrix}\end{aligned} \qquad (5.3)$$

Therefore, the PCT expansion of equation (5.3) is the PCT expansion of two systems

$$\begin{aligned}\dot{\boldsymbol{x}}_n &= \frac{\int_\Omega \dot{\boldsymbol{x}}(\zeta_1,\ldots,\zeta_k) w\Phi_n d\zeta_1\ldots d\zeta_k}{\int_\Omega w\Phi_n d\zeta_1\ldots d\zeta_k} \\ \dot{\boldsymbol{x}}_{cn} &= \frac{\int_\Omega \dot{\boldsymbol{x}}_c(\zeta_1,\ldots,\zeta_k) w\Phi_n d\zeta_1\ldots d\zeta_k}{\int_\Omega w\Phi_n d\zeta_1\ldots d\zeta_k}\end{aligned} \qquad (5.4)$$

Let the system in forward loop be equation (4.2) and the feedback path be (4.3). Therefore, equation (5.3) becomes:

$$\begin{aligned}\begin{bmatrix}\dot{x}1 \\ \dot{x}2\end{bmatrix} &= \begin{bmatrix}a & -1 \\ 1 & b\end{bmatrix}\begin{bmatrix}x1 \\ x2\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix} ref \\ \boldsymbol{y} &= \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x1 \\ x2\end{bmatrix}\end{aligned} \qquad (5.5)$$

The first order PCT expansion of equation (5.5) is

$$\begin{aligned}\begin{bmatrix}\dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_0 \\ \dot{x}_1\end{bmatrix} &= \begin{bmatrix}a_0 & 0 & -1 & 0 \\ 0 & a_0 & 0 & -1 \\ 1 & 0 & b_0 & \frac{1}{3}b_1 \\ 0 & 1 & b_1 & b_0\end{bmatrix}\begin{bmatrix}x1_0 \\ x1_1 \\ x2_0 \\ x2_1\end{bmatrix} + \begin{bmatrix}1 \\ 0 \\ 0 \\ 0\end{bmatrix} ref \\ y &= \begin{bmatrix}1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}x1_0 \\ x1_1 \\ x2_0 \\ x2_1\end{bmatrix}\end{aligned} \qquad (5.6)$$

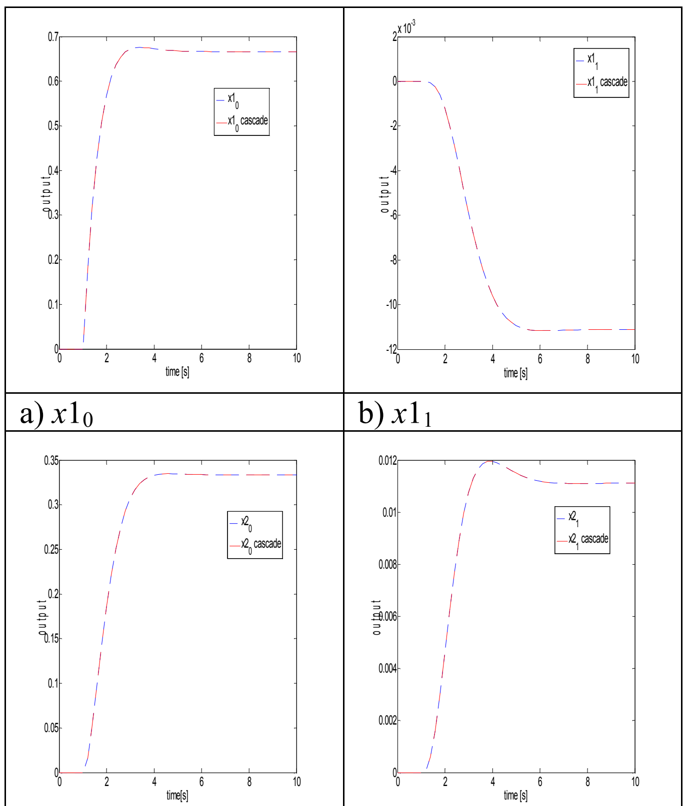
Using equation (5.3) the PCT expansion of the closed loop system can be found from PCT equation of system 1 (equation (4.10)), the system in the forward path and the PCT expansion of system 2 (equation (4.16)), the system in the feedback path.

$$\begin{aligned}\begin{bmatrix}\dot{\mathbf{x}}1_{pct} \\ \dot{\mathbf{x}}2_{pct}\end{bmatrix} &= \begin{bmatrix}\mathbf{A}_{pct} & -\mathbf{B}_{pct}\mathbf{H}_{pct} \\ \mathbf{G}_{pct}\mathbf{C}_{pct} & \mathbf{F}_{cpct}\end{bmatrix}\begin{bmatrix}\mathbf{x}1_{pct} \\ \mathbf{x}2_{pct}\end{bmatrix} \\ &\quad + \begin{bmatrix}\mathbf{B}_{pct} \\ \mathbf{0}\end{bmatrix} ref \\ \mathbf{y} &= \begin{bmatrix}\mathbf{C}_{pct} \\ \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}1_{pct} \\ \mathbf{x}2_{pct}\end{bmatrix}\end{aligned} \qquad (5.7)$$

The augment system becomes

$$\begin{aligned}\begin{bmatrix}\dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_0 \\ \dot{x}_1\end{bmatrix} &= \begin{bmatrix}a_0 & 0 & -1 & 0 \\ 0 & a_0 & 0 & -1 \\ 1 & 0 & b_0 & \frac{1}{3}b_1 \\ 0 & 1 & b_1 & b_0\end{bmatrix}\begin{bmatrix}x1_0 \\ x1_1 \\ x2_0 \\ x2_1\end{bmatrix} + \begin{bmatrix}1 \\ 0 \\ 0 \\ 0\end{bmatrix} ref \\ y &= \begin{bmatrix}1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}x1_0 \\ x1_1 \\ x2_0 \\ x2_1\end{bmatrix}\end{aligned} \qquad (5.8)$$

which matches equation (5.6).



a) $x1_0$ b) $x1_1$ c) $x2_0$ d) $x2_1$

Figure 8: Comparison of a first order expansion PCT outputs for the cascaded feedback blocks. Given a_0=-1, a_1=0.1, b_0=-2, b1=0.1.

6. CONCLUSION

Polynomial chaos theory is becoming a tool used to analyze and simulate the effects of uncertainty on a system. However, as the system increases and the number of uncertainty also increases, formulating a global PCT model become more cumbersome. This paper demonstrates that is it possible to propagate the uncertainty states using the signal coupling methodology. With this capability it will help the scalability problem using PCT methodology.

ACKNOWLEDGEMENT

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Biography

Anton H.C. Smith received his B.S. in Electrical Engineering in 2002, M.E. in 2004 and his PhD in 2008 all from the University of South Carolina. He is currently doing his post doctorate fellow at the University of South Carolina in the Virtual Test Bed group in the department of Electrical Engineering. His research interests are in digital control and robotics.

Antonello Monti received his his Ph.D from the "Politecnico di Milano" in 1994. From 1990 to 1994 he was with the research laboratory of Ansaldo Industria. In 1995 he joined the Department of Electrical Engineering of "Politecnico di Milano" as Assistant Professor. Since August 2000 he is Associate Professor at the EE Department of the University of South Carolina. He is author or co-author of more than 200 papers in the field of Power Electronics and Electrical Drives.

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Her current research interests are in the fields of integrated environments for distributed measurement and advanced simulation for monitoring and diagnostics of electrical systems. Dr. Ponci works with the Power and Energy research group at USC for the research and development of the electric ship, a project funded by the US Navy.