Design and Implementation of a Nonlinear Speed Control for a PM Synchronous Motor using the Synergetic Approach to Control Theory

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Abstract— This paper presents a novel nonlinear speed control for a permanent magnet synchronous motor (PMSM) using the synergetic approach to control theory (SACT). Recent research has reported that the PMSM is being increasingly used in high-performance applications, such as robots and industrial machines, which require speed controllers that provide not only accuracy and high performance, but also flexibility and efficiency in the design process and implementation. It has also been reported that the best approach to achieve high-performance in a PMSM drive is to consider the whole nonlinear motor dynamics in the controller synthesis. Many control schemes using varied nonlinear strategies have been presented; however, most of them are very complex to design and implement, even when they show good performance. We propose a nonlinear control scheme based on the SACT, which allows the designer to generate the required control laws by following a direct method. This scheme is also well suited for digital control implementation. An interesting outcome of the application of the SACT on the PMSM is the natural linearization and order reduction to a second-order system on the invariant manifold. This result allows the designer to set the desired dynamic behavior of the controlled system by selecting two system poles, as if employing a linear control strategy. The proposed SACT controller is implemented on a PMSM using a DSP-based platform and its performance is verified through the comparison of the experimental and simulation results.

I. INTRODUCTION

Technological advances in the electrical drives field have allowed the research and the industrial sector to set very demanding requirements concerning drive performance and efficiency. Among AC drives, the permanent magnet synchronous motor (PMSM) drives have demonstrated desirable features in a wide range of applications [1], [9].

A considerable amount of research has been devoted to investigate both linear and nonlinear control strategies that can provide efficiency, accuracy and robustness to motor drives [2]. For high-performance and precise speed control of a PMSM, linear control and linearization techniques have shown limitations while nonlinear control strategies have been demonstrated to be suitable solutions to deal with the PMSM’s nonlinearities [1], [2], [9]. In a PMSM, the nonlinear coupling between the stator winding currents and the rotor speed is the main nonlinearity that needs to be addressed.

A novel approach for the control of nonlinear systems, the synergetic approach to control theory (SACT), was introduced in [4] and [5]. Recent work has been reported on the application of the SACT to switching power converters [6], [7], [10], in which the high performance level, design simplicity and flexibility of SACT controllers have been demonstrated through both simulation and experiments. Furthermore, the feasibility of designing adaptive SACT controllers by selecting robust target manifolds was discussed in [11]. In that research, in order to achieve parameter variation insensitivity of the SACT controller, adaptive observers were avoided, because it was desirable to limit the computational burden of the controller implementation. For the same reason, in our specific application, the design and implementation of the SACT controller was carried out assuming constant experimentally estimated parameters of a low power motor drive.

This paper describes the design and implementation of a novel nonlinear SACT controller for a PMSM drive. In Section II the mathematical model of the PMSM in state-space form is presented. In Section III, the general procedure to design a SACT controller is explained. Section IV details the design steps followed to develop the SACT controller for our specific application. Finally, Section V explains the experimental and simulation results, considerations and procedures.

NOMENCLATURE

- \( i_{ds}, i_{qs} \): d- and q-axis stator currents
- \( \omega_s \): rotor electrical speed in angular frequency
- \( V_{ds}, V_{qs} \): d- and q-axis stator voltages
- \( R_s \): per-phase stator resistance
- \( L_s \): per-phase stator inductance
- \( K_t \): torque constant
- \( K_e \): back-EMF constant
- \( n \): number of pole pairs
- \( J \): moment of inertia of the rotor
- \( b \): damping coefficient
- \( \Phi_{PM} \): flux due to the rotor magnets
- \( \omega_r \): rotor electrical speed reference value
- \( \xi \): integral of the angular speed error
- \( \Psi_1, \Psi_2 \): macrovairables
- \( K_1, K_2, K_3 \): controller parameters
- \( T_c \): convergence time to the invariant manifold
- \( T_t \): load torque
- \( \bar{X} \): vector of state-variables
- \( f, g \): nonlinear functions
II. PMSM MODEL

In the synchronous d-q reference frame, the mathematical model [12], [8] of the PMSM is composed of a set of nonlinear state-space equations in the form

\[ \dot{x} = f(x) + \sum_{i=1}^{n} g_i(x)u_i. \]

The model system of equations follows:

\[
\begin{bmatrix}
\dot{i}_{sd} \\
\dot{i}_{sq} \\
\dot{\omega}_r
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{L_s} & 0 & \frac{1}{L_s} \\
\frac{1}{L_s} & -\frac{1}{L_s} & 0 \\
0 & 0 & -\frac{1}{J}
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
\omega_r
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} V_{sd} + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} V_{sq}
\]

(1)

State variables:
- \(i_{sd, sq}\): d- and q-axis stator currents
- \(\omega_r\): rotor electrical speed in angular frequency

System inputs:
- \(V_{sd, sq}\): d- and q-axis stator voltages.

where \(R_s\) is the per-phase stator resistance, \(L_s\) is the per-phase stator inductance, \(K_s\) is the torque constant \((K_s = K_e =\) back-EMF constant), \(n\) is the number of pole pairs, \(J\) is the moment of inertia of the rotor, \(b\) is the damping coefficient, and \(\Phi_{pm}\) is the flux due to the rotor magnets. Also, we consider the following relation \(K_s = \frac{2}{J} \Phi_{pm}\) [8].

The load torque \(T_l\) can be considered an unknown constant or inaccessible disturbance for the system shown in Eq. (1). Its adaptive estimation will be considered in the near future, but in this study, it is assumed to be zero.

The design of the SACT nonlinear controller, which is presented in detail in Section IV, requires that the PMSM third-order nonlinear system shown in Eq. (1) include the speed reference \(\omega_r\) to address the regulation problem. This variable can be introduced as part of the definition of the function \(f\) when the third-order system is augmented with a new state variable, \(\xi\), yielding a fourth-order system. Eq. (2) shows the resulting system with the two new variables:

\[
\begin{bmatrix}
\dot{i}_{sd} \\
\dot{i}_{sq} \\
\dot{\omega}_r \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{L_s} & 0 & \frac{1}{L_s} & 0 \\
\frac{1}{L_s} & -\frac{1}{L_s} & 0 & 0 \\
0 & 0 & -\frac{1}{J} & \frac{1}{J} \\
0 & 0 & 0 & -\frac{1}{J}
\end{bmatrix}
\begin{bmatrix}
i_{sd} \\
i_{sq} \\
\omega_r \\
\xi
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix} V_{sd} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} V_{sq}
\]

(2)

Augmented variables:
- \(\omega_r\): speed reference
- \(\xi\): integral of the angular speed error.

While augmenting the nonlinear system state-space representation to a fourth-order system may seem disadvantageous, Section IV and V demonstrate that in this specific application it is an appropriate and convenient design step.

III. SACT DESIGN

A few systematical steps are required to complete the synthesis of a SACT controller. The SACT is based on the principle of directed self-organization [3]. The SACT design procedure follows the analytical design of aggregated regulators (ADAR) method [4].

The procedure requires the model of the system to be in the state-space form

\[ \dot{x} = f(x) + \sum_{i=1}^{n} g_i(x)u_i \]

(3)

where a vector of state variables \(x\) is defined. Once the order of the system and the number of control channels \(u_i, i=1,2,\ldots,m\) has been specified, the state variables are combined such that the same number of macrovariables \(\Psi(x)\) as control inputs is created. These macrovariables can be created intuitively to fit the regulation objectives, as will be explained in the next section.

Each macrovariable \(\Psi(x)\) defines a manifold

\[ \Psi(x) = 0. \]

(4)

The set of manifolds form an invariant manifold as shown in Eq. (5):

\[ \Psi(x) = 0. \]

(5)

This invariant manifold describes the desired operating modes of the system, \textit{i.e.} the control objective. The system will be attracted by the invariant manifold according to the following functional equation:

\[ T \dot{\Psi} + \Psi = 0, T > 0 \]

(6)

which will force the system to exponentially converge to the invariant manifold, with a time constant \(T\).

The derivation of the control laws \(u_i\) is done after the macrovariables \(\Psi(x)\) are substituted into Eq. (6). Therefore, the control laws will be a function of all the system parameters, state variables and the convergence time to the invariant manifold \(T\).

Once the macrovariables have been defined, the SACT design procedure is merely algebraic; therefore, it is possible to automatize it. We have created a computer-aided control system design automation toolbox to streamline the SACT synthesis process and generate our control laws in an error-free manner. MATLAB Symbolic Math Toolbox was used for this purpose.

IV. PMSM SACT CONTROLLER

The method described in the previous section requires that we define the same number of macrovariables \(\Psi(x)\) as control channels in the system. Thus, \(V_{sd}\) and \(V_{sq}\) require the
definition of two macrovariables $\psi_i(\mathbf{x})$, which are a function of the state variables shown in Eq. (2).

The selected macrovariables for our case are shown in Eq. (7):

$$
\psi_1 = i_{sd}
$$

$$
\psi_2 = K_1(\omega_e - \omega^*) + K_2i_{sq} + K_3\xi
$$

(7)

where $K_1$, $K_2$, $K_3$ are controller parameters.

This selection is not arbitrary, since the physical reason can be deduced by inspection of the macrovariables.

The first macrovariable, $\psi_1$, which defines a manifold $\psi_1 = 0$, ensures the convergence of the d-axis current $i_{sd}$ to zero on the invariant manifold. In this way, we achieve rotor speed control by controlling only the q-axis current $i_{sq}$.

The second macrovariable, $\psi_2$, reveals the importance of the introduced state variable, $\xi$. In Eq. (2) the state variable $\xi$ is the integral of the rotor electrical speed error,

$$
\xi = \int (\omega_e - \omega_e^*) dt.
$$

(8)

Thus, combining two of the terms of macrovariable $\psi_2$, i.e. $K_1(\omega_e - \omega_e^*) + K_2i_{sq} = \int (\omega_e - \omega_e^*) dt$, we obtain a proportional-integral-like (PI) action that can be interpreted as negative of the q-axis current reference $-i_{sq}^*$ in a decoupled speed-torque control scheme. In this way, if $K_1 = 1$, then $\psi_2 = K_1i_{sq} - i_{sq}^* = i_{sq} - i_{sq}^*$ is the negative of the q-axis current error. On the invariant manifold, the q-axis current is expected to follow the reference value; hence both speed and torque control are achieved by enforcing the system to operate on the invariant manifold.

Having defined the macrovariables shown in Eq. (7), the functional equation $\psi_1 = 0$, $\psi_2 = 0$, $T > 0$ of the SACT can be applied, and the control laws $V_{sd}$, $V_{sq}$ can be derived by algebraic manipulation.

This systematic design procedure has been implemented to be automatically executed by a customized toolbox developed using the MATLAB Symbolic Math Toolbox.

The control laws $V_{sd}$, $V_{sq}$ shown in Eqs. (9) and (10) are a function of the motor and the controller parameters, including the convergence time to the invariant manifold $T$.

$$
V_{sd} = (TR_i i_{sd} - T\omega_e i_{sd} L_1 - i_{sd} L_1) / T
$$

(9)

$$
V_{sq} = -(-TK_1 J R_i i_{sq} - TK_2 J \omega_e i_{sq} L_1 + 0 - TK_2 J K_1 \omega_e + TK_i n^2 L_i K_1 i_{sq} - TK_i n L_i T_i + + TK_i L_i J \omega_e - TK_i L_i J \omega_e + - K_i L_i J \omega_e^* + K_3i_{sq} L_1 J + K_3\xi J)(JK_2)
$$

(10)

The SACT controller directs the system to the invariant manifold $\psi_1(\mathbf{x}) = 0$. Once on the invariant manifold, the closed-loop equations become

$$
\begin{bmatrix}
0 i_{sd}
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
- K_1 K_2 n^2 J K_1 & K_2 J & K_2 J & K_2 J & i_{sq}
0 & n^2 K_1 & K_1 & K_1 & K_1 & i_{sq}
0 & J K_1 & K_1 & K_1 & K_1 & \xi
0 & - K_3 J & K_3 J & K_3 J & K_3 J & \xi
0 & - K_3 J & K_3 J & K_3 J & K_3 J & 0
\end{bmatrix}
\begin{bmatrix}
0 & K_1 n^2 J & K_2 J & K_2 J & i_{sq}
0 & K_1 n^2 J & K_2 J & K_2 J & i_{sq}
\end{bmatrix} T_i.
$$

(11)

The eigenvalues of the system shown in Eq. (11) are

$$
\lambda_i = 0
$$

$$
\lambda_2 = 0
$$

$$
\lambda_3 = \frac{n}{2JK_2} (-K_1 K_2 n + \sqrt{K_2^2 K_2^2 n^2 - 4JK_2 K_2 K_2})
$$

(12)

$$
\lambda_4 = \frac{n}{2JK_2} (-K_1 K_2 n - \sqrt{K_2^2 K_2^2 n^2 - 4JK_2 K_2 K_2})
$$

It can be observed from Eq. (12) that two of the four eigenvalues are zero. This suggests that the system dynamics depend only on eigenvalues $\lambda_3$ and $\lambda_4$.

Two important outcomes are deduced from Eqs. (11) and (12). First, the system becomes linearized on the invariant manifold, i.e. the closed-loop system equations are linear. Second, the dynamics of the closed-loop system can be interpreted as a second-order dynamics. We conclude that the SACT controller reduced the system to a second-order linear system, with a zero steady state speed error.

Eigenvalues $\lambda_3$ and $\lambda_4$ can be written in the complex conjugate form as shown in Eq. (13).

$$
\lambda_{3,4} = \frac{n}{2J} \left( \frac{K_1}{K_2} \pm j \sqrt{4J^2 \frac{K_1}{K_2} K_2 - \left( \frac{K_2}{K_1} \right)^2 K_2^2 n^2} \right)
$$

(13)

After the convergence time $T$ has been specified ($T = 0.01$ sec), the controller parameters $K_1$, $K_2$, and $K_3$ can be obtained in closed form. On the invariant manifold, the system has two complex conjugate poles as shown in Eq. (13), which the designer can specify conveniently to achieve the desired behavior of the second-order system. We define $X$ and $Y$ as shown in Eq. (14).

$$
X = \frac{K_1}{K_2} \quad Y = \frac{K_1}{K_2}
$$

(14)
Selecting the system eigenvalues for a damping ratio $\zeta = 0.9$ and a natural frequency $\omega_n = 1000 \text{ rad/} \text{sec}$, we obtain the desired eigenvalues $\lambda_{1,4} = -1000 \pm 10j$. Solving the simultaneous equations for $X$ and $Y$ in Eq. (13), we get the values shown in Eq (15):

$$\lambda_{1,4} = -1000 \pm 10j, X = 0.0181, Y = 9.037$$

$$\therefore K_1 = 0.0181, K_2 = 1, K_3 = 9.037.$$ 

In this way, the SACT controller parameters $K_1, K_2$ and $K_3$ are calculated and the control is completely designed.

V. SIMULATION AND EXPERIMENTAL RESULTS

Fig. 1 shows the control block diagram of the drive. The control feedback is composed of four variables which are acquired from the plant: three phase stator currents $i_a, i_b$ and $i_c$, and the rotor angular speed $\omega_e$. The SACT controller block receives six inputs: d-q reference frame stator currents $i_{sd}$ and $i_{sq}$, rotor angular speed $\omega_e$, speed reference or set-point $\omega^*_e$, integral of the speed error $\xi$ and load torque $T_l$ ($T_l = 0$ for our current objective). The SACT controller computes the control laws and outputs two voltage levels in the d-q reference frame $V_{sd}$ and $V_{sq}$. The Park and inverse-Park transformation blocks shown in Fig. 1 provide the mathematical means to convert stator currents and voltages from the abc to the d-q reference frame. The space-vector pulse width modulator SV-PWM provides the duty cycles needed to control the 3-phase inverter that will actuate on the plant.

Fig. 2 shows the experimental test setup, with all the physical components used in the implementation of the complete control system.

Simulations were performed using MATLAB®/Simulink and the implementation was done using the dSpace DS1104 R&D platform.

The dSpace DS1104 R&D platform is a DSP-based controller board which provides a Simulink-compatible control desk. The controller board interfaces with the real plant (i.e. PM BLAC motor Pittman model 34X1 and PM50 power module board) to obtain the feedback signals from the motor and provide the controller outputs to the power stage.

The power stage module consists of a 15Vdc bus and a 3-phase inverter, which uses MOSFET transistors with switching frequency up to 20 KHz.

Both inputs and outputs of the system are specified through a Simulink schematic, in which the SACT controller is implemented using an S-function written in C code for the sake of compatibility.
TABLE I
DATASHEET SPECIFICATIONS OF PM BLAC MOTOR
PITTMAN MODEL 34X1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator Resistance per-phase</td>
<td>2.625 Ω</td>
</tr>
<tr>
<td>Stator Inductance per-phase</td>
<td>0.23e-3 mH</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>24.9e-3 N·m/A</td>
</tr>
<tr>
<td>Back-EMF constant</td>
<td>24.9e-3 N·m/A</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
<tr>
<td>Rotor Inertia</td>
<td>9.0e-7 Kg·m² (3.55e-5)</td>
</tr>
<tr>
<td>Damping Constant</td>
<td>1.2e-4 Nm/(rad/s)</td>
</tr>
<tr>
<td>Reference Voltage</td>
<td>19.1 V</td>
</tr>
<tr>
<td>No-load Current</td>
<td>0.072 A</td>
</tr>
<tr>
<td>Max Current</td>
<td>1.16 A</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>8000 RPM or 837.7 rad/sec</td>
</tr>
<tr>
<td>Peak Current</td>
<td>3.64 A</td>
</tr>
</tbody>
</table>

Table I shows the datasheet values for the PMSM (a.k.a. PM BLAC) Pittman motor (model 34X1) used for the lab experiment. This motor has low power and small dimensions. For this reason, its parameters—especially its mechanical parameters—require experimental estimation.

The moment of inertia $J$ and the damping constant $b$ of the rotor were estimated by injecting a pulse in the isolated q-axis current loop. The purpose of this experiment is to calculate $J$ and $b$ from time constants present in the speed transient produced by the $i_{sq}$ pulse injection in the rotor mechanical state-space equation

$$J \frac{dw}{dt} + bw = n^2 K_e i_{sq}, \text{ assumed } T_i = 0. \quad (16)$$

The experimental values of $J$ and $b$ are shown in parentheses in Table I.

Fig. 3 shows the controller performance through the transient behavior of the d-q stator currents.

Figs. 4, 5 and 6 show the comparison between the experimental and simulation results of the speed, and phase currents and voltages, which are in good agreement.

VI. CONCLUSION

This research work has investigated a novel nonlinear speed control for a permanent magnet synchronous motor (PMSM) using the synergetic approach to control theory (SACT). In this paper, both the design and implementation of the SACT controller for this application have been explained in detail. The SACT requires the creation of macrovariables (i.e. functions of the state variables of the
system) to define a set of manifolds that form an invariant manifold. The SACT principle of system self-organizing states that the system will converge to the invariant manifold, which is created such that our control objectives are met. The macrovariables specified for the SACT design in this study have been shown to be successful in the PMSM application. The simple and systematic nature of the SACT design procedure allows the designer to generate the required control laws by following a direct automatable method. We also demonstrate that the nonlinear control laws generated by the SACT procedure are suitable for digital control implementation, due to the simplicity of their online calculation. The main theoretical outcome of the use of the SACT to control a PMSM is the natural closed-loop system order reduction and linearization that the SACT produces on the invariant manifold. This result allows the designer to set the desired dynamic behavior of the controlled system by selecting two system poles, as if employing a linear control strategy (i.e. pole placement). The satisfactory performance is verified through the comparison of the experimental and simulation results.

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