A Decentralized Information Filter for the State Estimation in Electrical Power Systems

Gabriele D’Antona\textsuperscript{1}, Antonello Monti\textsuperscript{2}, Ferdinanda Ponce\textsuperscript{2}
\textsuperscript{1} Dept. of Electrical Engineering – Politecnico di Milano
\textsuperscript{2} Dept. of Electrical Engineering - University of South Carolina
P.za L. da Vinci, 32, 20133 Milano, Italy
Phone: +39-02-23993706, Fax: +39-02-23993703, Email: gabriele.dantona@polimi.it

Abstract – This paper shows the performance of a state estimator based on a decentralized information filter applied to an electric power system for avionic and naval applications. The main characteristics of this type of approach are firstly the increased computational efficiency, due to the parallel distribution of the computational burden among the local estimators, secondly an increased reliability of the estimator due to the distribution of the resources and thirdly the scalability of the estimation system.

Keywords – Distributed information filter, electric power system state estimation.

I. INTRODUCTION

For a reliable and efficient management of electrical power systems a real time state estimator of interconnected power networks is a key tool. Traditionally a state estimator for electric power systems has been conceived as an element of a central control system receiving as input measured data gathered from the network and stored in a central data base. On the basis of the state estimate decisions are taken in the central control system concerning the power system economy, quality and security [8,9].

In order to enhance the performance and reliability of the processing effort required from a control system a decentralized approach to the problem can be taken into consideration. This perspective, leading to the concept of a decentralized control system (vs. the central control system), brings together new paradigms especially related to (decentralized) decision and estimation functions, both ancillary to the control systems [10,11,12].

In this paper we will focus our attention to the decentralized state estimation functionality, with reference to highly reliable applications concerning electric power systems on board of ships and aircrafts.

A fully decentralized system is defined in [14] as a structure in which all information is processed locally, and no central processing site arises. More precisely a decentralized estimator is characterized by tree constraints [15]:

1. the absence of a central data assimilation processor;
2. the absence of a common communication facility: local estimator nodes can communicate on node-to-node basis;
3. estimator nodes do not have any global knowledge of the estimator network topology.

In [16] the authors proposed a solution to the decentralized data assimilation and estimation based on a distributed Kalman filter (DKF).

In this paper we propose a new solution to the estimation problem relying on an algebraic equivalence of the Kalman filter expressed in terms of measures of information, namely, the information filter. [13,14,15]. A comparison with the performance obtained with a DKF will reported.

II. PROBLEM FORMULATION

In the following we will consider a power system grid composed by the interconnection of \( N \) subgrids. The estimation task will take place in static condition, i.e. in quasi steady-state system behavior (only the dynamics of the electrical loads will be considered, while the dynamics of the electrical power converter will be neglected).

Considering a discrete time linear power system model described in the standard linear form:

\[
x(k) = A\cdot x(k-1) + B\cdot e(k-1) + w(k-1)
\]  

where \( x(k) \) is the state (independent voltages and currents in reactive elements of the grid) at time \( k \), \( e(k) \) the driving input vector and \( w(k) \) the model noise input modeled as a Gaussian i.i.d. (independent identically distributed) random process with zero mean and \( E[w(k)w(j)^T] = \delta_{kj}Q(k) \).

The power system is observed measuring branch voltages and currents modeled according to the following observation equation, derived on the basis of Ohm’s and Khirchhoff’s laws:

\[
y(k) = C\cdot x(k) + D\cdot e(k) + v(k)
\]  

where \( y(k) \) is the observation vector at time \( k \) and \( v(k) \) is the associated noise modelled as Gaussian i.i.d. random sequence with \( E[v(k)v(j)^T] = \delta_{kj}R(k) \).

The conventional Kalman filter generate estimates \( \hat{x}^i(k) \) of the state with covariance matrix \( P^i(k) \) according to the following algorithm:

Initialization:

\[
x^i(0) = x_0 \quad P^i(0) = P_0
\]  

...
Prediction:

\[ x^p(k) = A x^p(k-1) + B e(k-1) \]
\[ P^p(k) = A P^p(k-1) A^T + Q(k-1) \]

Measurement assimilation:

\[ x^i(k) = x^p(k) + K [y(k) - C x^p(k) - D e(k)] \]
\[ P^i(k) = (I - K(k)C) P^p(k) \]

with

\[ K(k) = P^p(k) C^T [C P^p(k) C^T + R(k)]^{-1} \]

The information filter is obtained recasting the Kalman filter equations in terms of two new variables [15] denoted the information matrix \( Y(k) \) and the information state \( y(k) \):

\[ Y(k) = [P^i(k)]^{-1} \]
\[ y(k) = [P^i(k)]^{-1} x(k) \]

The information filter algorithm is given by

Initialization:

\[ y^i(0) = (P_o)^{-1} x_0 \quad Y^i(0) = (P_o)^{-1} \]
\[ y^i(k) = L(k)y^i(k-1) + Y^i(k) B e(k-1) \]

Measurement assimilation:

\[ y^i(k) = y^i(k) + C^T R(k)^{-1} [y(k) - D u(k)] \]
\[ Y^i(k) = Y^i(k) + C^T R(k)^{-1} C \]

with

\[ L(k) = Y^i(k) C \cdot Y^i(k-1)^{-1} \]

As a consequence, for the applications here considered, we can assume that the loads are fed by power converters and then it is natural to consider these elements as the separation boundary between each subgrid. Considering the \( i^\text{th} \) subgrid we have:

Estimator input:
- measurement data vector \( y_i(k) \): bus voltages;
- breaker status vector and network model: binary data concerning the switch and circuit breaker positions and defining the network topology (slowly changing with time);
- parameter vector \( e_i(k) \): parameters characterizing the subgrid components, such as the electromotive forces or the voltage generators (slowly changing with time).

Estimator output:
- state vector \( x_i(k) \): independent voltages and currents in reactive elements of the grid.

Further, a number of sensors are considered to take the local observations \( y_i(k), i = 1, \ldots, M \). The process and observation models are thus obtained partitioning equation (1) and (2):

\[ x_i(k) = A_i x_i(k-1) + B_i e_i(k-1) + w_i(k-1) \]
\[ y_i(k) = C_i x_i(k) + D_i e_i(k) + v_i(k) \]

The partitioning in (1) and (2) is obtained with the procedure described in [16]. In particular, the electrical powers system.
is decomposed into $N=4$ subgrids, which can be classified as described below, with reference to the equivalent network shown in figure 2:

- 2 generators subgrids: R1-L1 and R2-L2.
- 2 loads subgrids: R3-L3-C3 and R4-L4-C4.

The behavior of the whole system is described by a set of 7 state variables, which, based on the decomposition introduced above, can be classified as shown in figure 2.

Fig.2 – State variable for the simplified equivalent network for the zonal system.

The observation random vector $y(k)$ can be partitioned with reference to figure 3.

Fig.3 – Output variable for the simplified equivalent network for the zonal system.

III. THE DECENTRALIZED STATE ESTIMATOR

In [16] we proposed a solution to the estimation problem based on a DKF, founded on the following three stages:

- local estimation
- communication
- assimilation of measurements data from other nodes.

With a similar approach also the information filter can be decentralized, leading to the following algorithm for the decentralized information filter (DIF):

Initialization:

$y_i^i(0) = (P_0)^{-1} \cdot x_0 \quad Y_i^i(0) = (P_0)^{-1}$

(19)

Prediction:

$Y_i^F(k) = \{A_i \cdot [P_i^A(k-1)]^{-1} \cdot A_i^T + Q_i \cdot (k-1)\}^{-1}$

$y_i^F(k) = L_i(k) \cdot y_i^i(k-1) + Y_i^F(k) \cdot B_i \cdot e(k-1)$

(20)

(21)

Measurement assimilation:

$y_i^A(k) = y_i^F(k) + \sum_{j=1}^N \cdot i_j(k)$

(22)

$y_i^A(k) = Y_i^F(k) + \sum_{j=1}^N \cdot I_j(k)$

(23)

where $i_j(k)$ and $I_j(k)$ are the information associated with the observations in each subgrid:

$i_j(k) = C_j^T \cdot R_j \cdot (k)^{-1} \cdot [y_j^F(k) - D_j \cdot u_j(k)]$

(24)

$I_j(k) = C_j^T \cdot R_j \cdot (k)^{-1} \cdot C_j$

(25)

with

$L_j(k) = Y_j^F(k) \cdot C_j \cdot Y_j^A(k-1)^{-1}$

(26)

With these definitions the information filter has the advantage that the assimilation is computationally simpler than in the Kalman filter, at the cost of increased complexity in local prediction. As a matter of fact at each node the state estimate cannot be determined simply by a linear combination of the $N$ contributions from each decentralized estimator node since the innovations are correlated. For the information filter instead the state estimate has the simpler form (22) which is easier to decentralize in a network of estimator nodes.

IV. NUMERICAL RESULTS

Figure 4 shows the numerical results obtained implementing in matlab the DIF required for the estimation of the seven state variables. As in [16] we have considered a sampling period $T_s = 1$ ms and a transmission period $T_t = 50$ ms.

In the simulation we have considered that at $t = 5$s the emf $E_2$ drops from 90V to 50V.

Considering the state variables and the measured data for this example, it results that the state variable $x_2$ is not observable. This result is confirmed by the simulation results shown in figure 4. In the numerical simulation we have also
considered that the estimator of the third subsystem was faulty and its data were not available to the other estimators.

The implementation of the simplified models of the DIF in each subsystem has been implemented considering the sparsity of the full transition matrix $A$ representing the dynamics of the whole electrical network. In the information filters we have considered only the most important coupling between state variables as in [16].

![Graphs showing state estimates](image1.png)

Fig. 4 – State estimate obtained by a DIF for the electrical network shown in figure 2. The offset between the estimated and actual current $x_2$ is due to the unobservability of the current in the loop $L'6-L'7-L''7-L''6$.

Figure 5 shows in detail the estimate of $x_3$ and $x_4$ during the transient occurring at $t=5$ s. The dotted line shows in figure 4 and 5 the true value of the state.

V. CONCLUSION

The paper reports the results of numerical simulations of a state estimator implemented using a distributed information filter for an electrical power system. The state estimation obtained with such a filter is optimal in case of a linear system with Gaussian i.i.d. noise. The estimate obtained with the DIF with asynchronous and incomplete assimilation is suboptimal but still very good in the simple example simulated in this work. The advantage of the DIF is that the transmission of new information can be done asynchronously only when the amount of information, measure with its entropy, is larger than a fixed threshold level. This new working procedure as not been implemented yet, but will be the objective of a future further development of this research.

![Graphs showing state estimates zoomed](image2.png)

Fig. 5 – Estimated states $x_3$ and $x_4$ zoomed at the transient instant $t=5$ s.
VI. REFERENCES


