Modeling and Stability Analysis in Multi-Converter Systems including Positive Feedforward Control

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Abstract- A Positive feedforward control technique is proposed to improve phase margin in multi-converter applications without physical hardware modification and with little reduction of system bandwidth. The positive feedforward control is realized by combining it with the conventional negative feedback control. The positive feedforward controller acts like an active filter presenting an impedance with positive real part at high frequencies while at low frequencies the negative feedback controller provides tight output voltage control. To provide controller design criteria, small-signal models by g-parameter representation are presented. Stability of the whole system with interactions between subsystems is analyzed based on the impedance criterion. As an illustrative example, the stability improvement by the positive feedforward control of a buck converter with an input filter is demonstrated by simulation in frequency domain. Experimental results including a constant power load connected in parallel to the input filter are also presented.

I. INTRODUCTION

In multi-converter systems, it is well known that the interaction between subsystems, even though each subsystem is individually well-designed and standalone stable, may cause an instability problem of the whole system [1]-[4]. It is also well known that input and output impedances of a switching converter are modified by control action. Typical examples are input voltage feedforward control and output voltage feedback control. A general representation for a converter in multi-converter applications is shown in Fig. 1. Impedance $Z_S$ is a lumped representation of the source system as seen at the input terminal of the converter, and $Z_L$ is a lumped representation of the load system as seen at the output terminal of the converter. An input voltage feedforward controller $G_{CFF}$ and output voltage feedback controller $G_{CFB}$ are also shown in Fig. 1.

Fig. 1. A regulated converter with the lumped output and input impedances of source and load subsystems.

In this work we examine the effect of input filter interaction, i.e., the frequently negative effect on system stability of the introduction of a non-zero source impedance $Z_S$, and propose a control structure that can improve system stability. The proposed approach is to add to the conventional output-voltage negative-feedback control an input-voltage positive-feedforward control, which improves stability margin by controlling the converter like an active filter in a certain frequency range. This active approach should be compared with conventional passive approaches such as power filter damping or the addition of decoupling capacitors at the converter input. In the filter damping approach a properly designed damping circuit is added to the input filter. This suppresses the resonant peak of the output impedance of the input filter enough to avoid the interaction [5]. One disadvantage of these passive approaches is the increase in system cost, size and weight due to the added passive components. Another disadvantage is that the modified input impedance may cause EMI issues. Finally, an increase in bus capacitance may cause inrush problems and may be otherwise undesirable [6].

Unlike those passive approaches, the proposed active approach using positive feedforward control is particularly attractive in that the stability improvement is achieved by control action only with no hardware modification of the physical system. Furthermore, the positive feedforward control stabilizing effect may be tuned online using an adaptive control approach to compensate for variations of the source impedance.

By utilizing both the positive feedforward and feedback control actions, the controller can regulate both the input and output characteristics of a converter. Obviously, there are trade-offs between what input and output characteristics can be achieved. By adjusting the control actions, it is possible, within limits, to tailor the desired input and output terminal characteristics of the converter. In other words, the positive feedforward control regulates input port characteristics of the converter at high frequencies so that it acts like an active filter, while the negative feedback control tightly regulates the output voltage at low frequencies so that it behaves as a conventional feedback control system.
II. SMALL-SIGNAL MODELING OF A CONVERTER

![Converter diagram](image)

The small-signal model for the positive feedforward control is obtained by substituting in (1) for the duty cycle the expression \( \hat{d} = -G_{CFB}\hat{v} \). The result is:

\[
\begin{bmatrix}
\hat{i}_{in} \\
\hat{v}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_{in\ FB}} & G_{id\ OL} & G_{id\ OL} \\
G_{vg\ OL} & G_{vd\ OL} & -Z_{out\ OL}
\end{bmatrix} \begin{bmatrix}
\hat{v}_g \\
\hat{d} \\
\hat{i}_{load}
\end{bmatrix}
\]  

where FB denotes feedback control. The feedback loop gain can be defined as \( T_{FB} = G_{CFB}G_{id\ OL} \). The transfer functions in (2) are given by

\[
\begin{align*}
Z_{in\ FB} &= \frac{1}{Z_{in\ OL} + \frac{1}{1 + T_{FB}}} \\
Z_{out\ FB} &= \frac{Z_{out\ OL}}{1 + T_{FB}} \\
G_{vg\ FB} &= \frac{G_{vg\ OL}}{1 + T_{FB}} \\
G_{ii\ FB} &= G_{ii\ OL} + \frac{G_{id\ OL}Z_{out\ OL} T_{FB}}{G_{vd\ OL} 1 + T_{FB}} \\
Z_{N\ vd\ OL} \text{ in (3) represents the input impedance for the case of ideal feedback control, i.e., for a feedback controller that perfectly regulates the output voltage. This impedance has a negative real part at low frequency and can cause system instability [5].}
\end{align*}
\]

The small-signal model for the negative feedback control system can be obtained by substituting in (1) for the duty cycle the expression \( \hat{d} = -G_{CFB}\hat{v} \). An equivalent feedforward control loop gain can be defined as \( T_{FF} = G_{CFB}G_{id\ OL} \). Notice that \( T_{FF} \) is not dimensionless and has units of admittance.

\[
\begin{bmatrix}
\hat{i}_{in} \\
\hat{v}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{Z_{in\ FF}} & G_{ii\ FF} \\
G_{vg\ FF} & -Z_{out\ FF}
\end{bmatrix} \begin{bmatrix}
\hat{v}_g \\
\hat{i}_{load}
\end{bmatrix}
\]  

In (7) the subscript FF denotes feedforward control. The transfer functions in (7) are:

\[
\begin{align*}
Z_{in\ FF} &= \frac{1}{Z_{in\ OL} + T_{FF}} \\
Z_{out\ FF} &= Z_{out\ OL} \\
G_{vg\ FF} &= G_{vg\ OL} + \frac{G_{vd\ OL} T_{FF}}{G_{id\ OL}} \\
G_{ii\ FF} &= G_{ii\ OL}
\end{align*}
\]

Equation (8) shows that, for a properly designed positive feedforward controller, the input impedance is dominated by the feedforward control loop gain \( T_{FF} \) at high frequencies. The simulation results presented later in the paper show why
this is a desirable design choice. Therefore, the converter input impedance at high frequencies is determined by the positive feedforward control.

Finally, for the combined positive feedforward and negative feedback control system, we can substitute in (1) the positive feedforward control. The transfer functions in (12) are:

\[
\begin{align*}
Z_{\text{out, FFFB}} &= Z_{\text{out, FB}} + \frac{1}{1+TFB} \\
G_{\text{VF, FFFB}} &= G_{\text{VF, FF}} + \frac{1}{1+TFB} \\
G_{\text{FF, FFFB}} &= G_{\text{FF, FB}}
\end{align*}
\]

In (12) the subscript FFB denotes the feedforward and feedback control. The transfer functions in (12) are:

\[
\begin{align*}
G_{\text{FF, FFB}} &= \frac{G_{\text{FF, FB}}}{1+TFB} \\
G_{\text{VF, FFB}} &= G_{\text{VF, FF}} + \frac{1}{1+TFB} \\
G_{\text{FF, FFB}} &= G_{\text{FF, FB}}
\end{align*}
\]

It is observed from (8) and (13) that the input impedance in the combined control is dominated by the negative feedback control at low frequencies and by the positive feedforward control at high frequencies. Unlike the negative feedforward control [8]-[10], the positive feedforward control has large audio susceptibility as shown in (10) but it is suppressed at low frequencies by large feedback control loop gain as shown in (15). Therefore, by the combined control, the output port of the system is regulated by typical negative feedback control at low frequencies and the input port of the system is regulated by the positive feedforward control at high frequencies. In particular, since the real part of the input impedance using the positive feedforward control becomes positive at high frequency, it can behave like an active filter providing input filter damping.

III. STABILITY ANALYSIS

Stability for the interconnected system can be assessed by the so-called Middlebrook impedance criterion [11]-[13]. In general, the interconnected system can be divided into a source subsystem and a load subsystem. The transfer function of the whole system is then described as

\[
G_{T12} = G_1G_2 \frac{Z_o}{Z_{in}} = T_{MLG} \frac{Z_{in}}{Z_{in}}
\]

where \( G_1 \) and \( G_2 \) are the transfer functions of the source and load subsystems respectively. If the source and load subsystems are standalone stable, the stability of the whole system can be analyzed by applying the Nyquist criterion to the minor loop gain (MLG), \( T_{MLG} \), which is the impedance ratio of the output impedance of the source subsystem, \( Z_o \), and the input impedance of the load subsystem, \( Z_{in} \).

IV. POSITIVE FEEDFORWARD CONTROLLER DESIGN

It is important to point out that the feedforward control commonly used in the control of switching converters is substantially different from the approach here proposed. Feedforward control is typically used to compensate for input voltage variations and provide tighter control of the output voltage [8]-[10]. As a result, the feedforward gain \( G_{FF} \) is negative at low frequency and the feedback input impedance has a negative real part at low frequency. Therefore, we can call the conventional feedforward control negative feedforward control. The feedforward control here proposed, which we will call positive feedforward control, has a positive feedforward gain, its goal is to stabilize the input port voltage rather than the output port voltage, and it tends to give an input impedance with a positive real part.

The minor loop gain for the source interaction with the positive feedforward and feedback control is given by

\[
T_{MLG, ZS} = \frac{Z_S}{Z_{in}}
\]

where \( Z_S \) is the output impedance of the source subsystem. The difference from the feedback control case (3) is the appearance of a third term dependent on the feedforward control loop gain. Examining the minor loop gain expression (18) we can make some important observations. The stability of the whole system is typically degraded by the second term due to the feedback action. The minor loop gain at low frequencies (for \( T_{FB} \gg 1 \)) can be approximated as

\[
T_{MLG, ZS} = \frac{Z_S}{Z_{N, OL}}
\]

which is identical to the feedback control case at low frequencies. Similarly at high frequencies (for \( T_{FB} \ll 1 \)) the minor loop gain can be approximated as

\[
T_{MLG, ZS} = \frac{Z_S}{Z_{in, OL}} + Z_STFF \approx Z_STFF
\]

In order to improve the stability of whole system, the third term in (18) should be properly designed so that it has higher gain and has a positive real part at high frequencies compared with the feedback control term.

V. SIMULATION

As an illustrative example, a buck converter with an input filter that represents the source subsystem is considered as
shown in Fig. 4, which also gives the values of power stage passive components. The switching frequency is 100 kHz.

The based open loop transfer functions using g-parameter representation (1) for an ideal buck converter system are given in Table I. The positive feedforward and negative feedback controllers are designed using the proposed approach. The control loop gains for the feedforward and feedback control are compared in Fig. 5.

It is noted that at low frequencies, the magnitude of the feedback control loop gain is higher than that of the positive feedforward control loop gain, while at high frequencies, the magnitude of the positive feedforward control loop gain is higher.

The feedback controller bandwidth is 4.8 kHz and the phase margin is 52.8 degrees. However, if the effect of the input filter is taken into account, the feedback control bandwidth and phase margin are degraded to 1.4 kHz and 14 degrees respectively due to the input filter interaction. This can be seen in Fig. 6. Also, the system degradation due to the input filter interaction is evident in the Nyquist plot of the minor loop gain (17) as shown in Fig. 7.

The input filter interactions can be clearly seen by comparing the output impedance of the input filter and the input impedance of the buck converter system as shown in Fig. 8. Notice how the input impedance of the buck converter system is dramatically modified by adding the positive feedforward controller. Although the overlap region in the magnitude plot is increased (more interaction) for the combined control, the phase margin is noticeably increased, improving system stability. This is different from general input filter design techniques based on a simplified impedance criterion that looks at magnitude only [1], [5], [11].
Fig. 8. The interaction between the input filter and the buck converter system for the cases of the feedback control and the combined control.

Fig. 9. The Nyquist plot of the minor loop gain for the positive feedforward and feedback control including the input filter.

The improved stability due to the positive feedforward control can be verified by comparing the Nyquist plot of the minor loop gain of Fig. 7 and Fig. 9. The phase margin is increased from 14 to 71.3 degrees with a small reduction of the system bandwidth from 1.4 kHz to 1.1 kHz.

VI. EXPERIMENTAL VERIFICATION

For experimental validation, a buck converter with input filter stage is built as shown in Fig. 10. A constant power load stage is connected to the output of the input filter to represent the destabilizing effect of other converters sharing the same input voltage bus. The digital control is implemented using a Virtex-II pro LC board based on Xilinx XC2VP4 Virtex-II pro FPGA (XC2VP4-5FG456). A P160 A/D module (two 12 bits 53 MSPS channels) is used to sense input and output voltages. A digital controller is designed by using System Generator in Matlab environment and compiled by using Xilinx ISE software. The constant power load stage includes a current source consisting of an op-amp (OP42) and a MOSFET (IXFK 80N50P), and an analog divider (AD734) [14]. The bandwidth of the constant power load is approximately 200 kHz, larger than the converter control bandwidth.

The digital feedback controller is tuned to have 5 kHz bandwidth and 58 degrees of phase margin, taking into account damping effects due to non-ideal hardware components. The digital feedforward controller is designed as shown in Fig. 5. The input voltage is 40V and the output voltage is 24V. The switching frequency is 100 kHz.

A step load change from 3 \( \Omega \) to 6 \( \Omega \) is applied to the system to test the control dynamic response. Two cases are compared: the feedback only (FB) case and the feedforward and feedback (FFFB) case. Fig. 11 and Fig. 12 show the control response for the FB case and the FFFB case, respectively. The input filter voltage, exhibits an oscillation due to the interaction between the input filter and buck converter. In the FB case of Fig. 11 the oscillation lasts 4ms; in the FFFB case of Fig. 12, the oscillation is reduced to about 2ms due to the damping effect of the feedforward control. To show the effect of the positive feedforward control as a stabilizer in multi-converter system, a 160W constant power load is connected to the buck converter input as shown in Fig. 10, and it draws 4A current from the 40V supply. The constant power load, which represents other converters connected to the same input bus, has a destabilizing effect, as shown for the FB case in Fig. 13. Notice the more severe input voltage oscillations lasting about 7 ms. For the FFFB case of Fig. 14, the oscillation is reduced to the level in Fig. 12 (no constant power load). This demonstrates that the FFFB controller can play a stabilizer role in multi-converter system with many constant power loads.
The positive feedforward control combined with the negative feedback control acts as a conventional feedback control at low frequency and as an active filter to improve input port stability at high frequency. An adaptive control approach can be used for the case of varying source or load subsystems/parameter uncertainties.

The stability improvement results in limited sacrifice of the system bandwidth and no hardware modification of the physical system. Furthermore, the proposed controller can act as a stabilizer in parallel connected multi-converter systems.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation under grant ECS-0348433.

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