Abstract—The inability of linear controls to predict dynamic behavior of nonlinear system leads to chaotic behavior of the system and sometimes could result in a system collapse. The advantage of synergetic control design is that it results in an analytical control law that reduces the dynamic order of the system, compensates for system nonlinearity by a particular choice of coefficients, and ensures asymptotic stability of the closed loop system. Our paper presents generalized control strategies for the system of an arbitrary number of paralleled buck converters feeding a constant power load. Using these generalized control strategies we develop a family of control strategies for a particular system of two buck converters covering a range of operating regimes. We show the impact on system dynamics of several different coordination algorithms that control transition between strategies. Finally, we describe how including a current limiting feature into the control decreases electrical stress on the switching components and we put forward a load estimator that simplifies the control strategies.

I. INTRODUCTION

Paralleling power converters is a very powerful tool that is able not only to decrease component electrical and thermal stress and overall system weight but also to enable dynamic current sharing that promises further improvement of dynamics, efficiency, and reliability of the power conversion stage. Implementation of this promising capacity of dynamic current sharing heavily relies on abilities of control strategies. However, so far, dynamic current sharing has not been studied and implemented to the fullest mostly due to challenges that nonlinearity, multi-connectivity, high order dynamics, and time-varying nature of these systems pose for control design.

Traditionally, classical control theory attracts engineers due to the variety of control design approaches which were developed to use simple linear small-signal models [1,2]. These approaches design different control laws such as integral-proportional etc, that stabilize the system. However, due to operating the system only in the vicinity of the operating point with specified quality, these laws cannot control the system beyond the linear limits [3]. As a result, the inability of linear controls to predict dynamic behavior of nonlinear system leads to chaotic behavior of the system and sometimes could result in a system collapse [4].

On the other hand, the problem of control design for paralleled buck converters has been addressed by more advanced techniques such as sliding control [3], feedback linearization control [5], fuzzy logic control [6], and synergetic control [7,8,9]. While [3,5,6,7] have implemented active current sharing with fixed and equally assigned current ratings, we have used synergetic control theory [8,9] to develop general control strategies for the case of m-paralleled buck converters feeding a resistive load where current ratings can be assigned to arbitrary values at arbitrary times. Control strategies with fixed current sharing, aimed to equally distribute power losses and stress among system components, are not able to take into account changes in the system operating conditions such as degradation or overheating of the system components. On the other hand, variable current sharing control strategies provide additional control channels to account for the system operating conditions. Reference [9] introduced the capability of synergetic control for dynamic current sharing where converter current ratings can be allocated dynamically based on the operating conditions of the system.

Examples [8,9] show that synergetic control theory has made it possible to achieve a more generalized understanding of dynamic current sharing in the system of paralleled converters due to the fully analytical procedure for control design. This procedure uses a nonlinear model of the system and ensures global (or semi-global) asymptotic stability of the closed-loop system. Moreover, synergetic control induces self-organized motion in the system that takes control of dynamic degrees of freedom of the system and as a result reduces the dynamic order of the controlled system and simplifies analysis of the closed loop system behavior. Besides, analytical control is very convenient for microprocessor implementation.

In this research we continue our study of synergetic control theory started in [7,8,9]. In particular we expand the introduced in [9] concept of dynamic current sharing where converter current ratings can be allocated dynamically based on the operating conditions of the system for the case of m-paralleled buck converter system feeding constant power load. The objectives of this research are to explore closed loop behavior as well as to study the implementation option for dynamic current sharing in a system of two buck converters that is built using a low cost microprocessor.
The present paper presents generalized control strategies for the system of an arbitrary number of paralleled buck converters feeding a constant power load. Next, using these generalized control strategies we develop a family of control strategies for a particular system of two buck converters covering such different operating regimes as stand-alone converter, master-slave and democratic current sharing. We explore the impact on system dynamics and efficiency of different coordination algorithms that control the transitions among strategies. Finally, we propose implementation of a current limiting feature that decreases electrical stress on the switching components of the system and we put forward a load estimator that simplifies the control strategies.

However, this digest covers only the most significant data such as the system model; general control strategies; stability conditions; parameters of the built system; some simulation results, including impact of control law coefficients on system current sharing and transient response; and comparison of simulation and experimental data. Additional details will be presented at the conference and will be further published as follow-on paper.

II. SYNERGETIC CONTROL DESIGN

The system containing an arbitrary number of paralleled converters feeding a constant power load is presented in Fig. 1. In model development, we limit the description level for each component by their lossless subset. Similarly, we limit the description level of the load by keeping only its capacitive, resistive, and constant power components, while sudden change of the system load is accounted for by disturbance \( M(t) \). Finally, we restrict the description level for the system model by using the state space averaging technique [10] as a tool for model development.

The state space averaged model of the system presented in (1) is derived using the following assumptions: the system operates in a continuous conduction mode; switching occurs at a very-high switching frequency relative to the filter dynamics; parasitic effects are ignored; state variables are the averaged \( v_{cl} \) capacitor voltage and \( i_{li} \) inductor currents; it is sufficient to represent the constant power load as a dependent current source having value equals the power consumed by the load \( P_{load} \), divided by the output voltage \( v_{cl} \); a piece-wise constant current disturbance \( M(t) \) is sufficient to approximate variations of the system load and any variation of system parameters; the origin of the system model is at the specified steady state voltage; the zero order dynamics adequately represents the behavior of the sources \( E_v \).

\[
\begin{align*}
M' &= \eta \cdot v \\
v &= \frac{1}{C_1} \left( \sum_{i=1}^{m} i_{li} \right) - \frac{v + V_{ci,ext}}{R_{ext}} - \frac{P_{load}}{v + V_{ci,ref}} \left( v + V_{ci,ref} \right) \frac{\delta \cdot M}{C_1} \\
\dot{v}_1 &= \frac{1}{L_1} \left( v + V_{ci,ref} \right) + u_1 \\
\dot{v}_2 &= \frac{1}{L_2} \left( v + V_{ci,ref} \right) + u_2 \\
&\vdots \\
\dot{v}_m &= \frac{1}{L_m} \left( v + V_{ci,ref} \right) + u_m \\
\end{align*}
\]

(1)

where \( v \) and \( i_{li} \) are state variables;

\[
v = v_{cl} - V_{cl,ref}
\]

\( C_1 + C_2 + \ldots + C_m + C_{ext} \) is the total output capacitance of the system of converters;

\( M(t) \) is a current disturbance influencing the system;

\( d_{i} \) is the desired switch duty of the \( i \)th converter;

\( \delta \) is a non-dimensional coefficient that will be determined later based on stability of the closed loop system

\[
\dot{u}_i = \frac{d_{i} \cdot E_{di}}{L_i}, \quad i = 1, m
\]

The disturbance \( M(t) \) that is introduced into the system model (1), is a dependent current source that accounts for changes of the system load. Since in real life applications converter systems are usually influenced by sudden load changes, the disturbance \( M(t) \) is approximated by a piece-wise constant function and is incorporated into the model as an integral action (2). As a result of introduction of an additional dynamic coordinate \( M(t) \) into the system model (1), the system voltage becomes invariant to the system load resistance.

\[
\frac{dM}{dt} = \frac{(v_{cl} - V_{cl,ref})}{T_f \cdot R_d} = \frac{v}{T_f \cdot R_d}
\]

(2)

The primary goal of the control is to maintain a specified output voltage \( V_{cl,ref} \) even while the system is affected by the time varying disturbance \( M(t) \). Another important goal is to ensure proper sharing of current among the parallel-connected converters. In contrast to the designs in [3,5,6,7], with fixed and equal current sharing, at the design stage we do not make any particular assumptions about current sharing except that the converter currents relate linearly to each other, that is they each supply some constant fraction of the load current. The capability for arbitrary current sharing is implemented by incorporating into the control design requirements additional variables (controls) that affect the interaction among the converters in the closed loop system. Defining these coefficients at the control design stage results in fixed current sharing. However, assigning the coefficients online (during
system (operation) enables dynamic current sharing. The resulting control laws must ensure asymptotic stability of the closed loop system.

From a mathematical standpoint, synergetic control design [11] is based on a new method for generating control laws \( u(\psi_i) = u_1(i_1, i_2, \ldots, v_c) \) or feedback that directs the system from arbitrary initial conditions into the vicinity of manifolds (3) and then ensures asymptotically stable motion along these manifolds toward the end attractors. On these attractors the desired properties of the controlled system are guaranteed. In short, the synergetic control design procedure is as follows. For system (1) the designer’s specifications are formulated as a set of macro-variables (4) based on the system state variables. The number of macro-variables in the set equals the number of control channels.

\[
\psi_i(i_{1,1}, i_{1,2}, \ldots, v_{c1}) = 0
\]

(3)

\[
\psi_i = \psi_i(i_{1,1}, i_{1,2}, \ldots, v_{c1}), \quad i = 1, m
\]

(4)

Assuming that the desired evolution of the macro-variables is as in (5) and substituting the system model (1) in functional equation (5), results in asymptotically stable control which ensures the desired dynamic properties.

\[
T_i \cdot \psi_i = -\psi_i, \quad T_i > 0
\]

(5)

Defining a macro-variable as shown in (6) and solving the system on functional equations (5) with system model (1) in mind, yields the control laws presented in (7).

\[
\psi_i = a_{i1} + a_{i2} \cdot \psi_i + \sum_{j=1}^{m} a_{ij} \cdot i_{1,2} + \frac{a_{i1}}{(v+V_{cl,ref})^2}, \quad i = 1, m
\]

(6)

or \( \Psi = AX - B \) where \( \Psi = (\psi_1, \psi_2, \ldots, \psi_m)^T \)

\[
X = (i_{1,1}, i_{1,2}, \ldots, i_{1,m})^T
\]

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1m} \\
  a_{21} & a_{22} & \cdots & a_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & a_{m2} & \cdots & a_{mm}
\end{bmatrix}, \quad B = \begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_m
\end{bmatrix}
\]

\[
b_i = -a_{i1} \cdot M - a_{i2} \cdot \psi_i - \frac{a_{i1}}{(v+V_{cl,ref})^2}, \quad i = 1, m
\]

(7)

\[
U = A^+ \cdot G \quad \text{where} \quad G = (g_1, g_2, \ldots, g_m)^T, \quad i = 1, m
\]

\[
g_i = \frac{1}{\frac{1}{n} \sum_{j=1}^{n} a_{i1} \cdot R_{c1} \cdot C_i} \left( \sum_{j=1}^{n} \sum_{k=1}^{n} A_i \cdot C_i \cdot R_{cl,ref} \cdot \frac{1}{v+V_{cl,ref}} \right) + \frac{1}{\frac{1}{n} \sum_{j=1}^{n} a_{i1} \cdot R_{c1} \cdot C_i} \left( \sum_{j=1}^{n} \sum_{k=1}^{n} A_i \cdot C_i \cdot R_{cl,ref} \cdot \frac{1}{v+V_{cl,ref}} \right)
\]

(8)

Substituting the expression for \( g_i \) from (8) into (7) gives:

\[
u_i = \sum_{j=1}^{n} A(i,j) \cdot g_j, \quad i = 1, m
\]

(9)

where \( A^{-1}(i,j) \) is the \( i,j \)th element of inverted matrix \( A \).

According to synergetic control theory [11], control laws (9) ensure asymptotically stable convergence towards the specified manifolds (4)within (3-5)T and, as shown later, results in current sharing specified by coefficients in (6). The study of the closed loop system behavior on manifolds via manifold equations (3) and elimination of dependent variables results in the second order dynamic system presented in (10).

\[
v''_c + F_1 \cdot v'_c + F_2 \cdot v_c + \frac{F_3}{(v_c + V_{cl,ref})} = 0
\]

(10)

where

\[
F_1 = \frac{1}{R_{cl,c1}} \sum_{j=1}^{n} \sum_{i=1}^{n} A^{-1}(i,j) \cdot a_{i2},
\]

\[
F_2 = \eta \left( \sum_{j=1}^{n} \sum_{i=1}^{n} A^{-1}(i,j) \cdot a_{i1} - \delta \right)
\]

\[
F_3 = \left( \frac{1}{C_i} \sum_{j=1}^{n} \sum_{i=1}^{n} A^{-1}(i,j) \cdot a_{i1+1} \right) \cdot P_{load}
\]

Selection of coefficients \( a_{i+1} \) to satisfy equality (11) simplifies the stability conditions for the closed loop system as shown in (12).

\[
F_3 = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \right) \cdot P_{load} = 0
\]

(11)

\[
F_1 > 0, \quad F_2 > 0, \quad \delta > 0, \quad i = 1, m
\]

(12)

Hence, synergetic control design results in an analytical control law that ensures asymptotic stability of the closed loop system and compensates for system nonlinearity by a particular choice of coefficients. Since the coefficients of matrixes A and B in (6) are arbitrary numbers such that \( \det A \neq 0 \) and equality (10) are satisfied, the properties of current sharing in the system can be defined during its operation as shown later in the paper.

### Table 1

<table>
<thead>
<tr>
<th>NOMINAL VALUE OF SYSTEM PARAMETERS</th>
<th>Name</th>
<th>Parameter</th>
<th>Nominal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance of inductors</td>
<td>( r_{i1} = r_{i2} = r_{in} )</td>
<td>0.2 Ohm</td>
<td></td>
</tr>
<tr>
<td>Filter inductance</td>
<td>( L_{i1} = L_{i2} = L_s )</td>
<td>6.1 mH</td>
<td></td>
</tr>
<tr>
<td>Filter capacitance</td>
<td>( C_{i1} = C_{i2} = C_e )</td>
<td>3000 µF</td>
<td></td>
</tr>
<tr>
<td>Input voltage</td>
<td>( E )</td>
<td>12 V</td>
<td></td>
</tr>
<tr>
<td>Output voltage</td>
<td>( V_{C,ref} = U_0 )</td>
<td>4.0 V</td>
<td></td>
</tr>
<tr>
<td>Switching frequency</td>
<td></td>
<td>4.846 kHz</td>
<td></td>
</tr>
<tr>
<td>Time constants</td>
<td></td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>Integrator coef.</td>
<td></td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>Law coef. ( a_{i1} )</td>
<td></td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Law coef. ( a_{i2} )</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Law coef. ( A )</td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Load resistance</td>
<td></td>
<td>2 Ohm</td>
<td></td>
</tr>
<tr>
<td>Disturbance resistance</td>
<td></td>
<td>2 Ohm</td>
<td></td>
</tr>
<tr>
<td>Disturbance constant power</td>
<td></td>
<td>4 W</td>
<td></td>
</tr>
</tbody>
</table>

III. SYSTEM SIMULATION AND EXPERIMENTAL VERIFICATION

Based on the implementation of the general control strategy, different families of control strategies were then developed for the two-converter system, the parameters of which are shown in Table 1.

A series of simulations were conducted to study the system behavior and properties of dynamic current sharing under different control algorithms. It is shown that manifold coefficients define not only the quality of current sharing in the system but they also influence the system transients.
A. Current sharing in the controlled system

In practice, master-slave and democratic current sharing strategies are widely used in systems of paralleled converters. The major difference between master-slave and democratic current sharing is that master-slave sharing provides a fixed ratio of current sharing in transient and static regimes of operation, while democratic sharing has a variable ratio of current sharing during transients and a fixed ratio in the static regime of operation. The coefficients \( a_{ij} \) introduced into the macro-variables (3), are the control channels that influence the rules of interaction among paralleled connected converters. In contrast to designs in [3,5,6,7] where the converters’ current rating are fixed, change of these coefficients in control law (9) during operation will affect not only converters’ ratings but also some other properties of the system behavior. This section shows important details of the impact of macro-variable coefficients on current sharing in the system and illustrates that synergetic control laws provide power systems with dynamic management of responsibilities, including dynamic reconfiguration and allocation of current sharing.

\[
\begin{pmatrix}
a_{13} & a_{14} \\
a_{23} & a_{24}
\end{pmatrix}
\begin{pmatrix}
i_{11} \\
i_{21}
\end{pmatrix}
= \begin{pmatrix}
a_{11}M - a_{12}v - \frac{a_{15}}{V_{cl,ref}} \\
a_{21}M - a_{22}v - \frac{a_{25}}{V_{cl,ref}}
\end{pmatrix}
\tag{13}
\]

After transients are passed, the representing point of the two-converter system inevitably reaches the intersection of the manifolds (3). Hence, the motion of the system on the manifolds is defined by the equations (13).

![Fig. 2. Structure of the system feedback on the manifolds](image)

In systems with two parallel-connected converters, the only possible basic current sharing regimes are master-slave and democratic. When the system is on the manifolds, either of these can be achieved by a particular choice of the coefficients in (13).

In systems with two parallel-connected converters, the only possible basic current sharing regimes are master-slave and democratic. When the system is on the manifolds, either of these can be achieved by a particular choice of the coefficients in (13).

**Table 2**

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Manifold structure</th>
</tr>
</thead>
</table>
| Master-slave     | \[
\begin{pmatrix}
a_{13} & a_{14} \\
a_{23} & a_{24}
\end{pmatrix}
\begin{pmatrix}
i_{11} \\
i_{21}
\end{pmatrix}
= \begin{pmatrix}
a_{11}M - a_{12}v - \frac{a_{15}}{V_{cl,ref}} \\
a_{21}M - a_{22}v - \frac{a_{25}}{V_{cl,ref}}
\end{pmatrix}
\tag{13}
\] |
| Democratic       | \[
\begin{pmatrix}
a_{13} & a_{14} \\
a_{23} & a_{24}
\end{pmatrix}
\begin{pmatrix}
i_{11} \\
i_{21}
\end{pmatrix}
= \begin{pmatrix}
a_{11}M - a_{12}v - \frac{a_{15}}{V_{cl,ref}} \\
a_{21}M - a_{22}v - \frac{a_{25}}{V_{cl,ref}}
\end{pmatrix}
\] |

Master-slave current sharing occurs when the master converter maintains the output voltage and the slave converter supplies the current according to the rating assigned to it [12], which is usually defined for the slave converter by the master. On the other hand, democratic current sharing occurs when each of the two converters provides current in proportion to its current rating and each converter independently accounts for the output voltage error [13]. Table 4 summarizes the impact of the \( a_{ij} \) coefficients on the current sharing properties.

Table 2 gives important insights not only about the current sharing properties but also about the responsibility of each converter. For example, coefficients \( a_{11}, a_{14}, a_{23}, \) and \( a_{24} \) define what fraction of the current is allocated to each of the converters, coefficients \( a_{12} \) and \( a_{22} \) define which converter is responsible for managing errors in the output voltage, and coefficients \( a_{13} \) and \( a_{23} \) define which converter is responsible for managing constant power load impact on the system.

B. System transient response and experimental validation

A series of simulation experiments were conducted to study...
For experimental validation, a system of two buck converters was built using low cost microprocessor Atmega8535 Fig.5. This microprocessor is an 8-bit RISC microcontroller with two high frequency PWM channels and 8-channel 10-bit ADC with internal and external voltage references. The operation set of the processor supports 8x8 multiplication, which makes the micro-processor very suitable for control applications.

\[ \text{output voltage error } (a) \]
\[ \text{inductor current } (i_1) \]

Fig. 6 The closed loop system transient response of the two-converter system presented in Fig. 6, shows good data agreement for inductor current and output voltage error, which is for control applications.

\[ \text{inductor current transients} \]
\[ \text{output voltage error transients} \]

Fig. 6 The closed loop system transient response of the two-converter system a) output voltage error b) inductor current (i_1)

IV. CONCLUSION

This report presented a design of generalized control strategies for the system of an arbitrary number of paralleled buck converters a feeding constant power load. Using these generalized control strategies we developed a family of control strategies for a particular system of two-buck converter built around a low cost microcontroller ATMega8535. These control strategies cover a range of operating regimes. The transition among the regimes was made by a simple change of coefficients in the manifold equations (3). In this way, certain coefficients of the manifolds (3) define current sharing in the static or transient regimes of the system’s operation. Moreover, in a complex system in which extended flexibility is required, current sharing assignments can be allocated dynamically. In addition, we showed the impact on system dynamics of coordination algorithms that control the transitions among the strategies. Finally, we proposed implementation of a current limiting feature that decreases the electrical stress on the switching components of the system, and we put forward a load estimator that allows simplification of the control strategies.

As is evident from this report, synergetic control yields practical advantages by providing an analytical control design procedure. Hence, synergetic control theory gives tools for the development of more generalized control strategies that reduce the dynamic order of the system, compensate for system nonlinearity, and ensure asymptotic stability of the closed loop system. These control strategies enable more control of dynamic current sharing, improve the system efficiency, decrease stress on system components, and promote simplification of control strategies.

The conducted study identified directions for further implementation of synergetic control such as reducing the dependence of transient current sharing on system parameters and developing an efficient coordination algorithm that can minimize system losses.

ACKNOWLEDGMENT

The authors acknowledge the support of US ONR grant N00014-02-1-0623.

REFERENCES