A Nonlinear Model for Studying Synchronous Machine Dynamic Behavior in Phase Coordinates

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Abstract—A new model for studying synchronous machine dynamic behavior is developed and implemented in the Virtual Test Bed (VTB) simulation environment. The nonlinear synchronous machine model uses quantities in phase coordinates. The advantages of phase-domain modeling for electric machines are discussed. The new model was applied in the study of start-up transients of a synchronous motor. The simulation results are in agreement with those obtained from a standard d-q model, thus validating our model. This paper also demonstrates that symbolic modeling tool can greatly simplify the process of developing complex nonlinear device simulation models.

Index Terms — Synchronous Machine, Modeling and Simulation, Symbolic method.

I. INTRODUCTION

The importance of synchronous machines in electric power systems has been well recognized. They are highly nonlinear, complex electromechanical device, whose dynamic behavior directly impacts the performance and reliability of the power system network. Apart from providing the ultimate electricity source, they are also used to enhance the power factor through reactive power support. To analyze the dynamic behavior of the synchronous machines, an effective and accurate simulation model is desired. However, it is difficult to develop such a model since synchronous machine dynamics are highly nonlinear due to the heavily nonlinear stator self inductances, stator mutual inductances, and mutual inductances between the stator and rotor windings. Further, the model needs to account for dynamics involving electrical and mechanical domains.

One essential requirement for a simulation environment of large-scale systems consisting of many nonlinear components is high computational efficiency, which facilitates fast user-machine interaction. A key technique in achieving this is the use of an advanced network solver that works with Resistive Companion Form (RCF) models [1, 2]. A resistive companion form model is based on a linearized form of the component differential equations. Specifically, during every time step of the simulation, the RCF component model expresses the through variables as a linear function of the across variables. If the component is highly nonlinear, the time domain solver may perform several iterations within a single time step in order to provide the convergence to a solution.

The Virtual Test Bed (VTB) is a simulation environment that was developed for this purpose. The VTB architecture is described in details in reference [3]. One of the unique capabilities of the VTB is advanced graphical user interface and visualization capabilities resulting in a virtual prototyping environment where new component and system designs can be evaluated. For example, the VTB user can modify the system configuration and component parameters during the simulation and immediately observe the system response.

This paper focuses on the development of a phase-domain Resistive Companion Form (RCF) model of a synchronous machine within the Virtual Test Bed. The equations that describe the full nonlinear dynamics of a synchronous machine in the phase coordinates are quite complex. Developing the simulation model by manual manipulation of the equations, performing numerical integration, and obtaining the Jacobian matrix as required for the RCF model is tedious and error-prone at best, and quite nearly unmanageable. Hence, it is desirable to have a high-level model development tool that will automatically generate the simulation model from “raw” equations that can be simply entered. Such a symbolic tool is implemented in the Virtual Test Bed [4].

II. RESISTIVE COMPANION FORM MODELING TECHNIQUE

Consider a power system component with a number of terminals through which it can be interconnected to other components, as illustrated in Fig. 1. Each terminal has an associated across and a through variable. If the terminal is electrical, these variables are the terminal voltage with respect to a common reference and the electrical current flowing into the terminal, respectively.

Such a component can be described with a set of algebraic-integral-differential equations of the following general form:
In order to create an RCF model of this component, equation (1) is integrated over the simulation time step and then approximated by a first order expression. Assuming a simulation time step \( h \), the result is of the form:

\[
\begin{bmatrix}
    i(t) \\
    0
\end{bmatrix} =
\begin{bmatrix}
    f_1(v(t), v(t-h), i(t), i(t-h), y(t), y(t-h), t) \\
    f_2(v(t), v(t-h), i(t), i(t-h), y(t), y(t-h), t)
\end{bmatrix}
\]

(2)

where \( f_1, f_2 \) are arbitrary vector functions; \( i \) is a vector of through variables; \( v \) is a vector of across variables; \( y \) is a vector of internal state variables; \( u \) is a vector of independent controls. Note that the functions \( f_1, f_2 \) define two sets of functions as well as form of vectors \( \hat{y} \).

The through variables appear only in the external equations. Similarly, the device states are classified as external states \( v(t) \) (i.e. the across variables), and internal states \( y(t) \). Equation set (1) is consistent in the sense that the total number of states is equal to the total number of equations.

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    0
\end{bmatrix} =
\begin{bmatrix}
    f_1(v(t), v(t-h), i(t), i(t-h), y(t), y(t-h), t) \\
    f_2(v(t), v(t-h), i(t), i(t-h), y(t), y(t-h), t)
\end{bmatrix}
\]

(2)

where \( G \) is the Jacobian matrix:

\[
G = \begin{bmatrix}
    \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial v} \\
    \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial v}
\end{bmatrix}
\]

(3)

\( b_1, b_2 \) are vectors depending in general on past history values of through, across variables or internal states and on values of these quantities at time instant \( t \). Note that if system (1) is linear, vectors \( b_1, b_2 \) depend solely on past history values of across and through variables and internal states. Functions \( f_1, f_2 \) in (3) are obtained from the corresponding functions \( f_1, f_2 \) in equations for the component model (1) after application of suitable integration technique. The form of these functions as well as form of vectors \( b_1, b_2 \) depends on the chosen integration method. The most common integration methods that can be used are the trapezoidal rule and second order Gear’s method.

III. THE ADVANTAGES OF MACHINE MODELING IN PHASE COORDINATES

To simplify the model development, researchers have been trying to model synchronous machines in the d-q domain with the aid of reference frame transformation. However, the d-q modeling technique introduces undesirable effects in dynamic simulation of a power system. Specifically, these effects include instability of interfacing the d-q model to the rest of the power system network, difficulties in modeling harmonics and unbalanced operation, and inefficiency in simulating multi-machine network since the d-q and inverse d-q transformations have to be performed at each time step. Therefore, there is a considerable research interest in phase-domain modeling for electric machines [5]-[8]. Although several phase-domain models based on the trapezoidal integration and Thevenin equivalent circuit modeling technique have been proposed, none of them can facilitate a system-wide simultaneous simulation solution.

In a general power system simulation, unlike the conventional model in d-q coordinates, the phase-domain model can be directly incorporated into the overall network (no transformations are needed at each time step) to generate more accurate (no numerical instability or predictions for certain unknown variables are present) and more informative (simulation solutions of the unknown variables are more related to actual physical quantities) network simulation results.

Furthermore, the phase-domain model of the synchronous machine is especially advantageous for simulation of multi-machine power systems since there is no need for multiple reference frames. Such a model can accommodate asymmetries and/or unbalanced operating conditions and can be extended to model nonlinear saturation effects in synchronous machines.

IV. THREE-PHASE SYNCHRONOUS MACHINE MODEL IN PHASE COORDINATES

The dynamic equations of the electrical and mechanical subsystems of a synchronous machine are derived in this section. The synchronous machine can be viewed as a set of mutually coupled inductors (as illustrated in Fig. 2), which interact among themselves to generate the electromagnetic torque. Mathematical model can be derived from circuit analysis with the following assumptions: (1) space mmf and flux waves are sinusoidally distributed (neglecting the teeth and slots effects); (2) saturation, hysteresis, and eddy currents are neglected. Fig. 2 illustrates the stator and rotor windings of a synchronous machine: three phase stator windings and three phase rotor windings. The stator is Y connected; the rotor has field winding and D and Q damper windings acting along the \( d \)- and \( q \)- axes respectively, with \( d \)-axis pointing to the positive magnetic axis of the field winding. Note that all inductors are mounted on the same magnetic circuit and thus they are all magnetically coupled. The position of the rotor is
denoted with the angle \( \theta(t) \), which is the position of its positive direct axis with respect to the static phase \( a \) magnetic axis.

![Diagram of General synchronous machine as a set of mutually coupled windings](image)

Application of Kirchoff's voltage law, Kirchoff's current law, and Faraday's law to the circuit of Fig. 2 yields:

\[
v_{abc}(t) = R_{abc}i_{abc}(t) + \frac{d}{dt} \lambda_{abc}(t) + \Gamma v_c(t)
\]

\[
0 = i_a(t) + i_b(t) + i_c(t)
\]

\[
v_{\alpha\beta\gamma}(t) = R_{\alpha\beta\gamma}i_{\alpha\beta\gamma}(t) + \frac{d}{dt} \lambda_{\alpha\beta\gamma}(t)
\]

where

\[
v_{abc}(t) = \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix}^T
\]

\[
v_{\alpha\beta\gamma}(t) = \begin{bmatrix} v_a(t) & v_b(t) & v_c(t) \end{bmatrix}^T
\]

\[
i_{abc}(t) = \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix}^T
\]

\[
i_{\alpha\beta\gamma}(t) = \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix}^T
\]

\[
\lambda_{abc}(t) = \begin{bmatrix} \lambda_a(t) & \lambda_b(t) & \lambda_c(t) \end{bmatrix}^T
\]

\[
\lambda_{\alpha\beta\gamma}(t) = \begin{bmatrix} \lambda_a(t) & \lambda_b(t) & \lambda_c(t) \end{bmatrix}^T
\]

\[
R_{abc} = \text{diag}(r_a, r_b, r_c) = \text{diag}(r, r, r)
\]

\[
R_{\alpha\beta\gamma} = \text{diag}(r_a, r_b, r_c)
\]

\[
\Gamma = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T
\]

\( \lambda_{abc}(t) \) is the vector consisting of magnetic flux linkages of phase \( a, b, \) and \( c. \) \( \lambda_{\alpha\beta\gamma}(t) \) is the vector consisting of magnetic flux linkages of the field winding \( f, \) the \( D \)-damper winding, and the \( Q \)-damper winding.

In above equations, the magnetic flux linkages are complex functions of the rotor position and the electric currents flowing in the various windings of the machine. The magnetic flux linkages of the stator and rotor phase windings are:

\[
\begin{bmatrix} \lambda_{abc}(t) \\
\lambda_{\alpha\beta\gamma}(t) \\
i_{abc}(t) \\
i_{\alpha\beta\gamma}(t) \end{bmatrix} = \begin{bmatrix} L_{aa}(\theta(t)) & L_{ab}(\theta(t)) & L_{ac}(\theta(t)) & 0 \\
L_{ba}(\theta(t)) & L_{bb}(\theta(t)) & L_{bc}(\theta(t)) & 0 \\
0 & 0 & 0 & L_{rr} \\
0 & 0 & 0 & L_{mr} \end{bmatrix} \begin{bmatrix} i_{abc}(t) \\
i_{\alpha\beta\gamma}(t) \end{bmatrix}
\]

Note that the inductances in above equations are dependent on the position of the rotor, which is time varying. Thus these inductances are time dependent. This makes the overall system highly nonlinear and time-varying.

The electromagnetic torque is computed by differentiating the field energy function \( w_m(\theta) \) with respect to the rotor position \( \theta_m(t) \); i.e.,

\[
T_m(t) = \frac{d}{d\theta_m(t)} w_m(\theta_m(t))
\]

Specifically, the electromagnetic torque can be expressed as follows:

\[
T_m(t) = \left( i_{abc}(t) \right)^T \begin{bmatrix} \frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \end{bmatrix} i_{\alpha\beta\gamma}(t)
\]

Finally, the model of the mechanical subsystem of the synchronous machine can be given as follows:

\[
\begin{align}
T_m(t) &= J \frac{d\omega_m(t)}{dt} - \left( i_{abc}(t) \right)^T \begin{bmatrix} \frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \end{bmatrix} i_{\alpha\beta\gamma}(t) \\
\frac{1}{2} \left( i_{abc}(t) \right)^T \begin{bmatrix} \frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \\
\frac{\partial L_{\alpha\beta\gamma}(\theta_m(t))}{\partial \theta_m} \end{bmatrix} i_{\alpha\beta\gamma}(t) \\
\end{align}
\]

\[
\frac{d\theta_m(t)}{dt} = \omega_m(t)
\]

\[
\theta_m(t) = \omega_m(t) + \frac{2}{p} \delta(t) + \frac{\pi}{p}
\]

where \( J \) is the moment of inertia of the shaft; \( p \) is the number of poles; \( T_m(t) \) is the mechanical load torque; \( \omega_m(t) \) is the mechanical speed; and \( \theta_m(t) \) is the rotor position.

V. IMPLEMENTATION OF THE MODEL USING SYMBOLIC METHOD

The synchronous machine model equations are manipulated and rearranged according to the format of (1) to facilitate the RCF model generation. The rearranged phase-domain model for the synchronous machine is highly nonlinear and time-varying. There are 43 equations in total. The Jacobian matrix for this model has \( 43^3 = 1849 \) elements. Although some of the elements are zero, many elements still have to be evaluated. Apparently, manual calculation of the Jacobian matrix would result in long development time of the model and even longer debugging time, since errors inevitably would be present. Such complexity of the phase-domain model of the synchronous machine is the main obstacle to manual development of its RCF representation. Fortunately, a symbolic tool developed in VTB can facilitate RCF model generation of such a complex synchronous machine model. Computational efficiency of the automatically developed RCF
VI. D-Q MODEL OF A THREE-PHASE SYNCHRONOUS MACHINE

For the purpose of comparison, the d-q model for a synchronous machine is also derived and implemented in Matlab/Simulink. The synchronous machine model equations are as follow (expressed in the rotor reference frame):

\[
v_{qs} = R_q i_q + \frac{d\lambda_{qs}}{dt} + \omega \lambda_{qs} \tag{21}
\]

\[
v_{ds} = R_d i_d + \frac{d\lambda_{ds}}{dt} \tag{22}
\]

\[
0 = R_{qf} i_{qf} + \frac{d\lambda_{qf}}{dt} \tag{23}
\]

\[
0 = R_{df} i_{df} + \frac{d\lambda_{df}}{dt} \tag{24}
\]

\[
v_J = R_J i_J + \frac{d\lambda_J}{dt} \tag{25}
\]

\[
j \frac{d\omega}{dt} = T_s - T_{load} \tag{26}
\]

where fluxes linkages:

\[
\lambda_{qs} = L_{qs} i_q + L_{mqs} i_d \tag{27}
\]

\[
\lambda_{ds} = L_{ds} i_d + L_{mds} i_q + L_{md} i_{qf} + L_{md} i_{df} \tag{28}
\]

\[
\lambda_{qf} = L_{qf} i_{qf} + L_{md} i_d \tag{29}
\]

\[
\lambda_{df} = L_{df} i_{df} + L_{md} i_q + L_{md} i_{qf} + L_{md} i_{df} \tag{30}
\]

\[
\lambda_J = L_J i_J + L_{md} i_d + L_{md} i_{qf} \tag{31}
\]

\[
T_s = \frac{3}{2} \frac{\text{poles}}{2} \left[ L_{mqs} i_d i_q + L_{md} i_d i_{qf} + L_{md} i_q i_{df} + (L_{md} - L_{md}) i_{qf} i_{df} \right] \tag{32}
\]

Note that all quantities including variables and parameters are referred to the rotor side.

Descriptions of the symbols in above equations are as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_s</td>
<td>stator phase winding resistance</td>
<td>L_mqs</td>
<td>q-axis magnetizing inductance</td>
</tr>
<tr>
<td>R_{ds}</td>
<td>rotor q-axis damper winding resistance</td>
<td>L_{md}</td>
<td>stator d-axis winding inductance</td>
</tr>
<tr>
<td>R_{df}</td>
<td>rotor d-axis damper winding resistance</td>
<td>L_{md}</td>
<td>d-axis magnetizing inductance</td>
</tr>
<tr>
<td>R_{qf}</td>
<td>rotor field winding resistance</td>
<td>L_{qf}</td>
<td>rotor q-axis damper winding inductance</td>
</tr>
<tr>
<td>J</td>
<td>machine shaft inertia</td>
<td>L_{md}</td>
<td>rotor d-axis damper winding inductance</td>
</tr>
<tr>
<td>L_{qs}</td>
<td>stator q-axis winding inductance</td>
<td>L_{df}</td>
<td>rotor field winding inductance</td>
</tr>
</tbody>
</table>

VII. SIMULATION OF STARTUP TRANSIENTS OF A SYNCHRONOUS MACHINE

To test our model in VTB, a system consisting of a 3-phase voltage source, a synchronous machine, an excitation source, and mechanical load is simulated. The schematic diagram of the simulated system is illustrated in Fig. 3. The total simulation time is about 1.4 seconds. At a specific time instant (0.8 sec), the mechanical torque load of 50 Nm is applied to the shaft of the synchronous motor.

The parameters of the three-phase synchronous machine are as follows [9]:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prated</td>
<td>20 kVA</td>
<td>L_mqs</td>
<td>2.0 mH</td>
</tr>
<tr>
<td>(r_s)</td>
<td>0.1 (\Omega)</td>
<td>L_{md}</td>
<td>4.89 mH</td>
</tr>
<tr>
<td>(r_f)</td>
<td>0.016 (\Omega)</td>
<td>L_{qf}</td>
<td>2.79 mH</td>
</tr>
<tr>
<td>(r_D)</td>
<td>0.17 (\Omega)</td>
<td>L_{md}</td>
<td>4.48 mH</td>
</tr>
<tr>
<td>(r_Q)</td>
<td>0.17 (\Omega)</td>
<td>L_{md}</td>
<td>4.39 mH</td>
</tr>
<tr>
<td>poles</td>
<td>4</td>
<td>L_{qf}</td>
<td>2.91 mH</td>
</tr>
<tr>
<td>(L_{md})</td>
<td>4.1 mH</td>
<td>(J)</td>
<td>0.2 kgm(^2)</td>
</tr>
</tbody>
</table>

Simulation results obtained from the VTB are shown in the Fig. 4 through 6.
The same scenario as above is simulated using the d-q synchronous machine model implemented in Matlab/Simulink. The simulation results are shown in Fig. 7, 8, 9.
From the above simulations, the results from the VTB model in phase coordinates match closely to the results of standard d-q model, thus verifying the validity of the new model.

VIII. CONCLUSIONS AND DISCUSSIONS

A nonlinear time-domain simulation model of a synchronous machine was developed based on the actual phase quantities. Such a phase-domain model has many advantages for the dynamic simulation of a power system involving multiple machines under different operating conditions. The resulting model was implemented in a virtual prototyping simulation environment and validated by comparison to a standard d-q model. It is also demonstrated that a symbolic tool can greatly reduce the effort required to develop a complex simulation model.

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