

## A DECENTRALIZED STATE ESTIMATOR FOR NON-LINEAR ELECTRIC POWER SYSTEMS

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**Abstract - State estimators are an integrating part of traditional power systems. Usually state estimators for electric power systems have been conceived as elements of a central control system receiving as input measured data gathered from the network and stored in a central data base. Furthermore, the traditional structure does not usually base the estimation on the integration of a dynamic model. This view could be considered as a limiting factor for modern power systems on board of ships or aircrafts.**

**In order to solve both issues raised above the authors proposed in a recent publication to define the state estimation for a power system starting using a decentralized Kalman filter. Furthermore a novelty of the approach proposed concerns with the formalization of the estimation problem: in our approach the estimation is not exclusively based on the measurement data but also on a dynamic model of the power grid.**

**This paper shows the performance of a decentralized Kalman filter applied to an electric power system for avionic and naval applications.**

### INTRODUCTION

For a reliable and efficient management of electrical power systems a real time state estimator of interconnected power networks is a key tool. Traditionally a state estimator for electric power systems has been conceived as a an element of a central control system receiving as input measured data gathered from the network and stored in a central data base. On the basis of the state estimate decisions are taken in the central control system concerning the power system economy, quality and security [8,9].

In order to enhance the performance and reliability of the processing effort required from a control system a

decentralized approach to the problem can be taken into consideration. This perspective, leading to the concept of a decentralized control system (vs. the central control system), brings together new paradigms especially related to (decentralized) decision and estimation functions, both ancillary to the control systems [10,11,12].

In this paper we will focus our attention on the decentralized state estimation functionality, with reference to highly reliable applications concerning electric power systems on board of ships and aircrafts.

A fully decentralized system is defined in [14] as a structure in which all information is processed locally, and no central processing site arises. More precisely a decentralized estimator is characterized by three constraints [15]:

1. the absence of a central data assimilation processor;
2. the absence of a common communication facility: local estimator nodes can communicate on node-to-node basis;
3. estimator nodes do not have any global knowledge of the estimator network topology.

In this paper we propose a new solution to the estimation problem relying on a decentralized Kalman filter (DKF).

### PROBLEM FORMULATION

In the following we will consider a power system grid composed by the interconnection of N subgrids. The estimation task will take place in static condition, i.e. in quasi steady-state system behavior: only the dynamics of the electrical loads will be considered, while the dynamics of the electrical power converter will be neglected. As result of this assumption we can assume a linearized model for the non-linear time-variant power

electronic components. Depending on the bandwidth assumed for the power converter our hypothesis will require different time horizon in the analysis. An interesting aspect of analysis, not covered in this paper, is the definition of the limits of acceptability of this assumption that allows the transformation of the non-linear problem to a linear problem. Considering a discrete time linear power system model described in the standard linear form:

$$\mathbf{x}(k) = \mathbf{A} \cdot \mathbf{x}(k-1) + \mathbf{B} \cdot \mathbf{u}(k-1) + \mathbf{w}(k-1) \quad (1)$$

where  $\mathbf{x}(k)$  is the state (independent voltages and currents in reactive elements of the grid) at time  $k$ ,  $\mathbf{u}(k)$  the driving input vector and  $\mathbf{w}(k)$  the model noise input modeled as a Gaussian i.i.d. (independent identically distributed) random process with zero mean and  $E[\mathbf{w}(k)\mathbf{w}(j)^T] = \delta_{kj}\mathbf{Q}(k)$ .

The power system is observed measuring branch voltages and currents modeled according to the following observation equation, derived on the basis of Ohm's and Kirchoff's laws:

$$\mathbf{y}(k) = \mathbf{C} \cdot \mathbf{x}(k) + \mathbf{D} \cdot \mathbf{u}(k) + \mathbf{v}(k) \quad (2)$$

where  $\mathbf{y}(k)$  is the observation vector at time  $k$  and  $\mathbf{v}(k)$  is the associated noise modelled as Gaussian i.i.d. random sequence with  $E[\mathbf{v}(k)\mathbf{v}(j)^T] = \delta_{kj}\mathbf{R}(k)$ .

The conventional Kalman filter generate estimates  $\mathbf{x}^A(k)$  of the state with covariance matrix  $\mathbf{P}^A(k)$  according to the following algorithm:

Initialization:

$$\mathbf{x}^A(0) = \mathbf{x}_0 \quad \mathbf{P}^A(0) = \mathbf{P}_0 \quad (3)$$

Prediction:

$$\mathbf{x}^F(k) = \mathbf{A} \cdot \mathbf{x}^A(k-1) + \mathbf{B} \cdot \mathbf{u}(k-1) \quad (4)$$

$$\mathbf{P}^F(k) = \mathbf{A} \cdot \mathbf{P}^A(k-1) \cdot \mathbf{A}^T + \mathbf{Q}(k-1) \quad (5)$$

Measurement assimilation:

$$\mathbf{x}^A(k) = \mathbf{x}^F(k) + \mathbf{K} \cdot [\mathbf{y}(k) - \mathbf{C} \cdot \mathbf{x}^F(k) - \mathbf{D} \cdot \mathbf{u}(k)] \quad (6)$$

$$\mathbf{P}^A(k) = (\mathbf{I} - \mathbf{K} \cdot \mathbf{C}) \cdot \mathbf{P}^F(k) \quad (7)$$

with

$$\mathbf{K}(k) = \mathbf{P}^F(k) \cdot \mathbf{C}^T \cdot [\mathbf{C} \cdot \mathbf{P}^F(k) \cdot \mathbf{C}^T + \mathbf{R}(k)]^{-1} \quad (8)$$

The state estimation problem analyzed in this work is defined partitioning the power grid in interconnected subgrids.

The problem of partitioning a complex power system is not new and the definition of the more convenient point of separation is a critical task [1,2,3,4].

In this paper we focus on ship and aircraft applications. In the design of the All Electric Ship there

is a significant trend on restructuring the power system to adopt the so called DC Zonal Distribution. Interesting examples of application of this concept are reported in [5] or in [6].

An interesting characteristic of the DC Zonal System is the definition of natural points of separations given by the zone interface converters. Furthermore, the presence of a power converter at the connection point creates a dynamic decoupling between each zone and the rest of the system. An example of such topology is reported in figure 1 [7].

In the diagram it is possible to recognize the main elements: main source, interface converters, zonal load. For the purpose of this work we assume to model each zonal load together with its local converter as an independent system dramatically reducing the complexity of the model from the whole ship to a section of it. This solution is well justified by the topology of the power plant and it perfectly fits the structure of the observer proposed in this paper.

We could imagine more complex scenarios where we have multiple sources for the main distribution or also where the main source is an AC three-phase source. For the point of view of the modelling this would not change the approach: AC/DC converter would be considered in steady state and operating with a steady state duty-cycle not affected by the time evolution of the network. Furthermore, by using Park transformation the problem is formally reduced to the same problem studied here involving DC/DC converters.

For this set of reasons, we decided to work with a single DC source in order to maintain the complexity of the problem such that it is possible to intuitively understand the system evolution.

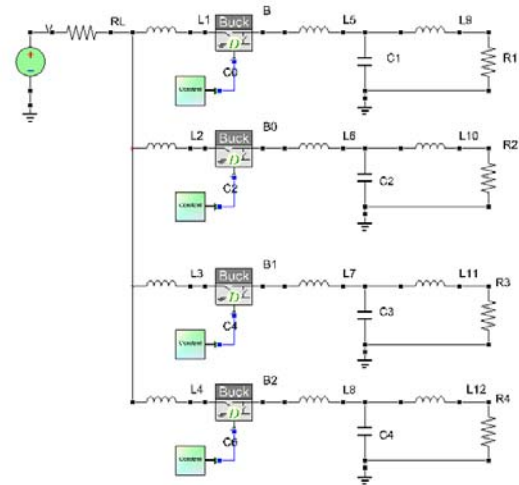


Fig. 1. A zonal system.

As a consequence, for the applications here considered, we can assume that the loads are fed by

power converters and then it is natural to consider these elements as the separation boundary between each subgrid. Considering the  $i^{th}$  subgrid we have:

Estimator input:

- measurement data vector  $\mathbf{y}_i(k)$ : bus voltages;
- breaker status vector and network model: binary data concerning the switch and circuit breaker positions and defining the network topology (slowly changing with time); (in this case we consider a single topology but the breaker topology could be added in more complex cases)
- parameter vector  $\mathbf{u}_i(k)$ : parameters characterizing the subgrid components, such as the electromotive forces or the voltage generators (slowly changing with time).

Estimator output:

- state vector  $\mathbf{x}_i(k)$ : independent voltages and currents in reactive elements of the grid

Further, a number of sensors are considered to take the local observations  $\mathbf{y}_i(k)$ ,  $i = 1, \dots, M$ . The process and observation models are thus obtained partitioning equation (1) and (2):

$$\mathbf{x}_i(k) = \mathbf{A}_i \mathbf{x}_i(k-1) + \mathbf{B}_i \mathbf{u}_i(k-1) + \mathbf{w}_i(k-1) \quad (9)$$

$$\mathbf{y}_i(k) = \mathbf{C}_i \mathbf{x}_i(k) + \mathbf{D}_i \mathbf{u}_i(k) + \mathbf{v}_i(k) \quad (10)$$

The partitioning in (1) and (2) is obtained with the procedure described in [16]. In particular, the electrical powers system is decomposed into  $N=4$  subgrids, which can be classified as described below, with reference to the equivalent network shown in figure 2:

- 1 generators subgrid representing the main distribution line
- 4 local converter subgrids

The behavior of the whole system is described by a set of 12 state variables, which, based on the decomposition introduced above, can be classified as shown in figure 2.

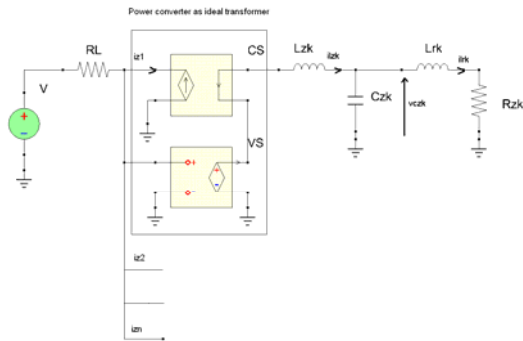


Fig.2 – State variable for the simplified equivalent network for the zonal system.

The observation random vector  $\mathbf{y}_i(k)$  can be partitioned with reference to figure 3.

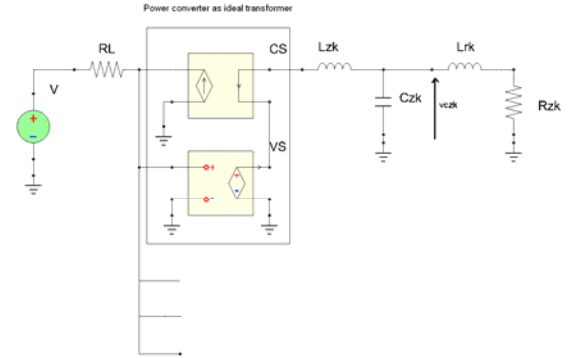


Fig.3 – Output variable for the simplified equivalent network for the zonal system.

With reference to Figure 2 it is reasonably simple to write the correspondent state equations for a single zone:

$$\begin{aligned} L_{zk} \frac{di_{zk}}{dt} &= v_{zk} - v_{czk} \\ C_{zk} \frac{dv_{czk}}{dt} &= i_{zk} - i_{rk} \\ L_{rk} \frac{di_{rk}}{dt} &= v_{czk} - R_{zk} i_{rk} \end{aligned} \quad (11)$$

Where we have that the power converter has been substituted by an ideal transformer with turn ratio equal to the duty cycle. Furthermore the inductance  $L_{zk}$  is obtained by reporting to the secondary of the transformer the inductance of the line representing the distribution between the main bus bar and the zone converter under analysis.

## THE DECENTRALIZED STATE ESTIMATOR

In [16] we proposed a solution to the estimation problem based on a DKF, founded on the following three stages:

- local estimation
- communication
- assimilation of measurements data from other nodes.

With a similar approach also the information filter can be decentralized, leading to the following algorithm for the DKF:

Initialization:

$$\mathbf{x}_i^A(0) = \mathbf{x}_{0i} \quad \mathbf{P}_i^A(0) = \mathbf{P}_{0i} \quad (12)$$

Prediction:

$$\mathbf{x}_i^F(k) = \mathbf{A}_i \mathbf{x}_i^A(k-1) + \mathbf{B}_i \mathbf{u}_i(k-1) \quad (13)$$

$$\mathbf{P}_i^F(k) = \mathbf{A}_i \mathbf{P}_i^A(k-1) \mathbf{A}_i^T + \mathbf{Q}_i(k-1) \quad (14)$$

Local measurement assimilation:

$$\mathbf{x}_i^A(k) = \mathbf{x}_i^F(k) + \mathbf{K}_i(k) \times [\mathbf{y}_i(k) - \mathbf{C}_i \mathbf{x}_i^F(k) - \mathbf{D}_i \mathbf{u}_i(k)] \quad (15)$$

$$\mathbf{K}_i(k) = \mathbf{P}_i^F(k) \mathbf{C}_i^T \mathbf{R}_i(k)^{-1} \quad (16)$$

Asynchronous distributed measurement assimilation:

$$\begin{aligned} \tilde{\mathbf{x}}_i^A(m) &= \mathbf{x}_i^A(m) + \\ &+ \tilde{\mathbf{P}}_i^A(m) \sum_{j \neq i} [\mathbf{P}_j^A(m)^{-1} \mathbf{x}_j^A(m) - \mathbf{P}_j^F(m)^{-1} \mathbf{x}_j^F(m)] \end{aligned} \quad (17)$$

$$\tilde{\mathbf{P}}_i^A(m)^{-1} = \mathbf{P}_i^A(m)^{-1} + \sum_{j \neq i} [\mathbf{P}_j^A(m)^{-1} - \mathbf{P}_j^F(m)^{-1}] \quad (18)$$

With these definitions the DKF gives in each node the same estimate as a Kalman filter if:

1. the distributed measurement assimilation time  $m$  is synchronous with the local sampling time  $k$ ;
2. if the local models are the same for each node and coincident with the global model, i.e if:

$$\mathbf{A}_i = \mathbf{A}, \mathbf{B}_i = \mathbf{B}, \mathbf{C}_i = \mathbf{C}, \mathbf{D}_i = \mathbf{D}, \mathbf{u}_i = \mathbf{u}$$

In our case we have considered a different situation since the global assimilation is asynchronous and the local model are all the same but are a simplified version of the global fully connected model.

## NUMERICAL RESULTS

Figure 4 shows the numerical results obtained implementing in matlab the DKF required for the estimation of the seven state variables. We have

considered a sampling period  $T_s = 0.1$  ms and a transmission period  $T_t = 5$  ms. The relative standard deviation of the measured data was  $10^{-3}$  p.u.

In the simulation we have considered that at  $t = 0.1$  s the voltage  $V_{cd}$  drops from 700V 70% of its value and at the same time it starts to rump up to 1400V in 0.1s.

Considering the state variables and the measured data for this example, it results that the state variable  $x_2$  is not observable. This result is confirmed by the simulation results shown in figure 4. In the numerical simulation we have also considered that the estimator of the third subsystem was faulty and its data were not available to the other estimators.

The implementation of the simplified models of the DIF in each subsystems has been implemented considering the sparsity of the full transition matrix  $\mathbf{A}$  representing the dynamics of the whole electrical network. In the information filters we have considered only the most important coupling between state variables as in [16].

Figure 5 shows in detail the estimate of  $x_1$  during the transient occurring at  $t=0.1$  s. The dotted line shows in figure 4 and 5 the true value of the state. In figure 5 is possible to note the effect estimate correction due to the global assimilation occurring every 5 ms. Figure 6 shows the same estimate without any global assimilation.

## CONCLUSION

The paper reports the results of numerical simulations of a state estimator implemented using a distributed Kalman filter for an electrical power system. The state estimation obtained with such a filter is optimal in case of a linear system with Gaussian i.i.d. noise. The estimate obtained with the DKF with asynchronous and incomplete assimilation is suboptimal but still very good in the example simulated in this work.

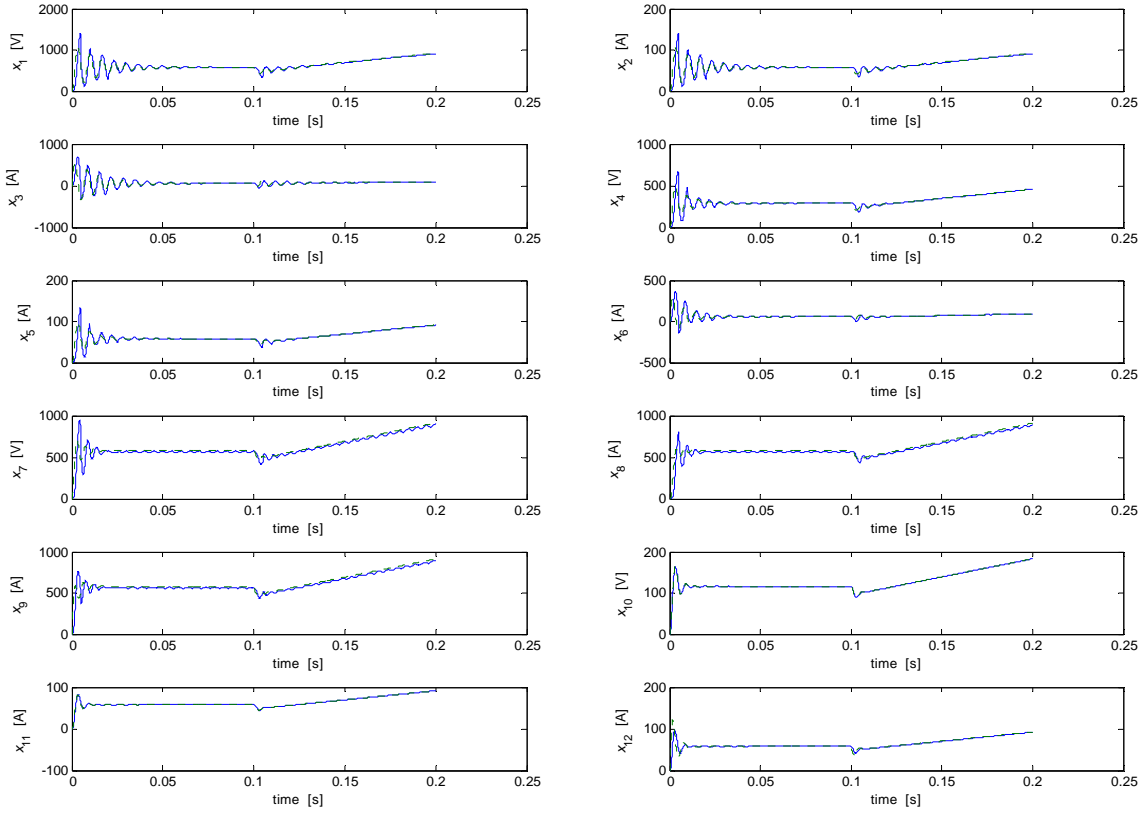


Fig.4 – State estimate obtained by a DKF for the electrical network shown in figure 2.

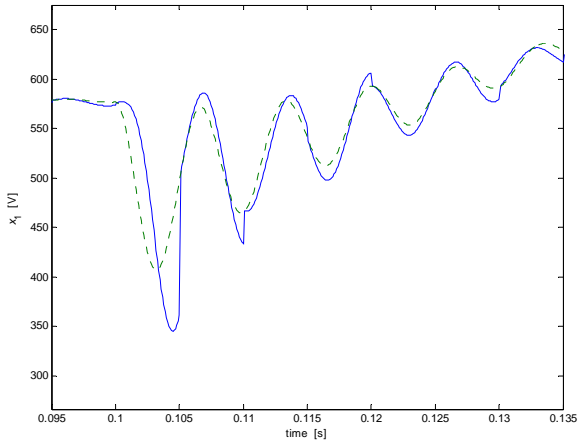


Fig.5 – Estimated state  $x_1$  zoomed at the transient instant  $t=0.1s$ .

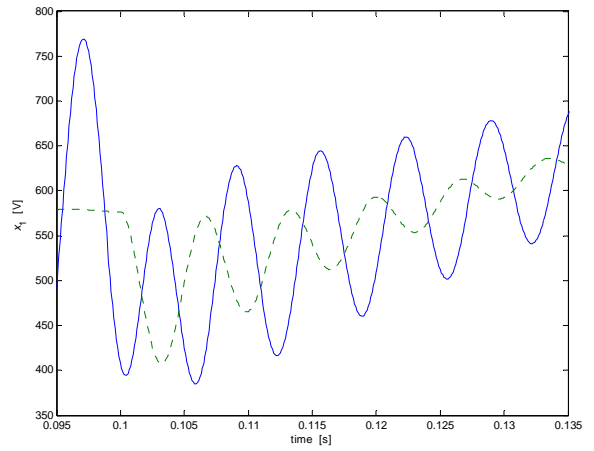


Fig.6 – Estimated states  $x_3$  and  $x_4$  zoomed at the transient instant  $t=5s$ .

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