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Discrete-Time Multi-Resolution Modeling of Switching Power Converters Using Wavelets

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Modeling of power electronics circuits presents significant challenges due to the non-linear time-varying behaviour of this kind of system. It is common practice to operate with families of models of different complexity depending on the analysis goal. In this paper we propose a wavelet-based approach as a unifying framework able to provide multi-resolution capabilities and consequently significant flexibility to the modeling process. The comparison with more classical approaches to power converter modeling demonstrates how this approach can be considered an extension of formalizations such as state-space averaging. Simulation results for both DC-DC and DC-AC power converters, including soft-switching configurations, are presented as sample applications. The discrete-time formulation makes this approach particularly suitable for design and analysis of converters operated with digital controllers.

Keywords: power electronics, wavelet transforms, modeling, power conversion

1. Introduction

Switching power converters are highly non-linear systems with widely different time constants. Different modeling approaches proposed in literature present various trade-offs between simplicity and speed of execution. The most commonly used physics-based modeling approaches, listed in order of increasing complexity and accuracy, are: POPI (power conservative: $P_o = P_i$) model [1], linearized state space averaging and state space averaging [2], discrete modeling [3], generalized state space averaging [4], and ‘exact’ modeling.

By far, the most commonly used modeling approach is state space averaging, in nonlinear form for large signal simulation, and in linearized form for control design and small-signal stability analysis. However, this method is only applicable to certain classes of converters. In fact, in the derivation of the model a so-called small ripple (or linear ripple) approximation is made. In essence the converter switching period is assumed to be much shorter than the system time constants. As a consequence, this method is not applicable to converters having large ripple components, such as resonant converters, line-commutated converters and converters operating at very low switching frequencies. The last two classes of converters include many FACTS (Flexible AC Transmission Systems) converters, which are of interest in multi-converter systems.

Discrete-time modeling is not very popular because it is rather inconvenient to interface the discrete model of each power converter with the continuous models of the rest of the system. With the increased use of digital control techniques this modeling approach is becoming more popular. An interesting approach to discrete time modeling is discussed by Maranesi [5] and then expanded by Maranesi and Riva [6].
Finally, the ‘exact’ modeling approach describes a converter as a time-varying system. These models are typically not applicable for control design purpose because they are difficult to analyze, and impractical for the simulation of the relatively large systems of interest, because they require simulation time steps much smaller than the switching period, and result in a very computationally expensive simulation. These models can also differ one from the other for the level of details adopted to describe the physics of the semiconductor devices [7, 8].

In generalized state space averaging the states are expanded in a truncated Fourier series over an interval of duration T, and the coefficients of the Fourier series are the new state variables [9]. The equations describing the system are then expressed as a function of these new variables. This method is an extension of state space averaging, with state space averaging as the particular case of the Fourier series truncated to the DC term. The accuracy of this method depends on the number of terms of the Fourier series, thus it can be determined by the user. Clearly, arbitrary waveforms can be accommodated by this method. In literature this method is called generalized state space averaging when applied to DC-DC converters and is sometimes called dynamic phasor method when applied to AC systems. In the latter case, the dynamic phasors are the coefficients of the Fourier series, which in steady state become standard phasors. In literature this method has been applied to FACTS and to synchronous and induction machines [14–18].

The use of wavelets, among the most recent developments, is a promising new approach to the modeling and simulation of power converters [10, 11, 13, 19, 20]. It has been shown that this method accurately predicts the behavior of the converter under both continuous and discontinuous conduction mode. Furthermore, it provides steady-state time domain waveforms and frequency-domain small-signal transfer functions that are useful for control design. Finally, it can be used for DC-DC as well as DC-AC converters [12]. The wavelet-based method shares with generalized state space averaging the desirable feature of being a variable-order method, allowing the possibility to obtain a variable degree of accuracy, depending on the order of the model that is chosen. While the order in generalized state space averaging is determined by selecting the truncation term for the Fourier series, in wavelet-based models the order is determined by the number of generations of wavelet to use. If a suitable mother wavelet is selected, an advantage of the wavelet approach is that fewer terms are needed to obtain a given desired accuracy, in comparison to the Fourier series. In other words, the wavelet representation proves to be a very compact representation of a given system. This property has been exploited more in general for the analysis of electrical circuits, as reported for example in the preliminary studies [21–23], then generalized in a more comprehensive framework by Barmada and Raugi [24]. While these approaches share the common idea of using wavelet for circuit solution, they do not focus specifically on the analysis of switching power converter, as instead in this work, as reported herein.

It should be clarified that various wavelet bases can be used to develop the proposed approach. In this paper the authors focus on the Haar wavelet basis, for its simplicity and its capability to describe step-wise waveform in a very compact way. This solution seems promising in particular for power electronics applications where the switching action creates piecewise constant waveforms.

The paper is organized as follows: Section 2 presents a brief introduction to wavelets, Section 3 describes the formalisms for application to power converters. Section 3 contains a set of application examples showing analysis for DC-DC and DC-AC systems. Section 4 expands the domain of analysis based on wavelet models, describing the significance of the eigenvalues in the wavelet domain.

2. Introduction to Wavelets

In this section the form of the Haar wavelet transform as is used throughout this paper is presented.

Firstly, we discuss some properties of wavelets that make them desirable in various applications and in particular in power electronics. Wavelets are typically used for time-frequency analysis. Time-frequency analysis is necessary when treating transient signals or signals with singularities. Unfortunately the time and frequency resolutions cannot be made independently good. Since the Heisenberg uncertainty principle holds, the choice of time and frequency resolution turns out to be a trade-off, and better time resolution results in a poorer frequency resolution and vice versa. Referring to the windowed Fourier transform, for example, it is intuitive to see that a window with a large support and consequently more concentrated frequency content, allows for a better frequency resolution but results in an inferior time resolution. Once the window is defined, the time-frequency resolution is fixed and it is identical at every frequency. Instead, the wavelet approach allows for more flexibility. With reference to the discrete wavelet transform, a wavelet base can be built in such a way that the resolution is different at different frequencies [32]. For example, a wavelet base may possess better frequency resolution at low frequency and better time resolution at high frequency. This structure is convenient if we want to precisely identify the fundamental frequency of an electrical signal and we want to identify with good time precision the presence of sudden glitches. The second desirable characteristic of wavelets is related to their multi-resolution analysis capability. The analysis performed with wavelets allows for the representation of a signal as the superposition of a coarser representation and subsequent finer details. The coarseness of the representation and the level of detail can be set as desired. Significant results based on wavelet analysis in the area of measurements and power systems are reported in several papers [25, 26, 29–31].
This orthogonal wavelet family allows for the decomposition of any $L^2(\mathbb{R})$ function into the sum of a coarse representation related to the scaling function, which plays the role of averaging function, and a sum of finer and finer details related to the wavelets. This is the core of multi-resolution analysis.

A convenient aspect of the decomposition calculation is that it only involves sums and differences in discrete calculations. The effect of the wavelet decomposition is that of performing subsequent filtering on the original signal, as mentioned above.

In our studies we deal with finite sequences of $N$ samples of the analyzed signal, $\{x(n/N)\}$, where $N = 2^J$. Uniform sampling in the interval [0,1] is carried out, yielding a sequence of $N$ samples univocally related to the Haar-sequence of $N$ wavelet coefficients. By ordering these sequences respectively in the column vectors of samples $x_N$ and coefficients $X_N$, the following matrix relation holds:

$$x_N = [H_N]^T X_N$$

(4)

where we have introduced the square matrix $[H_N]$, whose $N$ rows are made up of samples of ordered Haar-functions (where Haar-functions are $\phi(\cdot)$ and $\psi_{j,k}(\cdot)$ of the $n$-th term of the sequence of samples):

$$H_N[k][n] = \text{Haar} \left( k, \frac{n}{N} \right).$$

(5)

Due to the orthogonality of the Haar basis, $[H_N]^T [H_N] = [I_N]$, the following synthesis relation can be inferred:

$$X_N = [H_N]^T x_N.$$  

(6)

The Haar transformation matrix for the case of the use of four wavelet coefficients is reported here as an example:

$$[H] = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}.$$  

(7)

In what follows, the calculations will be expressed in matrix form, since this approach makes the implementation of the algorithms in computer-aided tools rather straightforward.

### 3. Wavelets and Switching Circuits

This section discusses the modeling of switching circuits through a discrete state-equation in the wavelet domain. Firstly the state variables are decomposed in the wavelet domain. The refinement of the representation can be pushed as far as desired, although it will be shown that few wavelet coefficients allow, in the cases presented here, for a satisfactory representation.

It is herein assumed that a state space description of the circuit is available. This formulation in many cases can be easily obtained by inspection of the circuit but it could
also be synthesized automatically starting from the models of the components. Procedures for automatic synthesis of state space models for circuits are reported, for example, by Wasyczuk and Sudhoff [27].

First of all, we formalize the representation of the transition of the state of the system from one switching period to the next in terms of the transition from time slot to time slot, within each switching period.

Let us consider a circuit switching with constant period Ts. The finer detail representation can not exceed the sampling period. This implies that, wishing to identify the details in terms of wavelet of N-th order, we need a sampling period equal to Ts/N. The sampling periods within the switching period will be called time slots.

Let us assume that the circuit switches between two linear configurations [28]. For each i-th configuration of the circuit, in this case with i = 1,2, it is possible to write the state-equations in the form:

\[
\dot{x}(t) = A_i x(t) + B_i u(t)
\]

where \(\dot{x}\) is the vector of the derivatives of the state variables, and \(x\) and \(u\) are vector of the state variables and vector of the input of the system respectively. Let us now discretize the model and adopt a zero-order hold approximation of the input. For a generic n-th time slot (where \(t = nTs/N\)) we will have:

\[
\dot{x}(n + 1) = F_i x(n) + G_i u(n)
\]

where:

\[
F_i = e^{A_i T_i / N} \quad G_i = \int_0^{T_i / N} e^{A_i \eta} d\eta B_i.
\]

For example, let us consider a system with only two possible different configurations. An F-G pair defines each configuration, so \(F_1\) and \(G_1\) are the matrices for the configuration number one and \(F_2\) and \(G_2\) for the configuration number two. The generic \(F_i\) matrix is a square matrix, whose size is equal to (number of state variables) \(x\) (number of state variables) and the generic matrix \(G_i\) is a rectangular matrix with size equal to (number of state variables) \(x\) (number of external inputs). Let us call \(T(n)\) a sample of the external input source.

Let us consider the simple case of a system with two linear configurations and a duty-cycle of 50%. In Figure 2 two switching periods are represented. For example, assuming the system is described by two state variables, \(x_1\) and \(x_2\), and referring to the period division in Figure 2, the transition between time slot \(n\) and time slot \(n + 1\) occurs according to the following equation:

\[
\begin{bmatrix}
    x_1(n + 1) \\
    x_2(n + 1)
\end{bmatrix} = [F_1] \begin{bmatrix}
    x_1(n) \\
    x_2(n)
\end{bmatrix} + [G_1][T(n)]
\]

We assume here that the input, in a certain time slot, takes the same value it had two time slots before, that is, \(T(n + 2) = T(n)\).
of a quantity \( m(k) \) the wavelet decomposition vector \( M_w \) is defined as:

\[
[M_w] = [H][M]
\]  

(15)

where \( H \) is a suitable matrix defined according to the selection of the base. An example of such matrix, for the Haar wavelet was reported in Section 2. Considering that the state vector \( X(k) \) is defined as composition of samples of all the state variable, we introduce a new transformation matrix \( H_w \), that can be applied to \( X(k) \), defined as a diagonal block matrix composition of the matrix \( H \):

\[
[H_w] = \begin{bmatrix}
[H] & 0 & 0 & 0 \\
0 & [H] & 0 & 0 \\
0 & 0 & \cdots & 0 \\
0 & 0 & 0 & [H]
\end{bmatrix}
\]  

(16)

where the number of blocks is equal to the number of state variables.

From a system theory standpoint, because the inverse transformation matrix \( H_w^{-1} \) exists and is equal to the transposed of \( H_w \), then the matrix \( H_w \) defines a similarity transformation for the system. Left multiplying the system model by the transformation matrix yields:

\[
[H_w]X(k + 1) = [H_w][F]X(k) + [H_w][G]U(k). 
\]  

(17)

Then, by substituting the following identities:

\[
X(k) = [H_w]^{-1}X_H(k) \quad U(k) = [H_w]^{-1}U_H(k) \quad (18)
\]

and knowing that the following identity holds, \([H_w][H_w]^T = [I]\), we conclude that the transformed set of equations has the following expression:

\[
X_H(k + 1) = [F_w]X_H(k) + [G_w]U_H(k) \quad (19)
\]

where \( X_H(k) \) and \( X_H(k + 1) \) are the vectors of the wavelet coefficients of the state variables during the \( k \)-th and \( (k + 1) \)-th period, \( U_H(k) \) is the vector of the wavelet coefficients of the input and \( F_w \) and \( G_w \) are the transition and state matrices in the wavelet domain. By applying the inverse of the Haar transformation, i.e. by left multiplying by the transposed of the \( H_w \) matrix, it is also possible to obtain the time evolution of the variables within the interval. The number of available time samples is normally equal to the number of wavelet levels adopted in the analysis. For example, if we stop the analysis only at the scaling function level, we obtain one value of each state variable in each switching period, and this corresponds to the average value of the state variable. The more details we add, the more values are reconstructed within each switching period. In other words the wavelet-based representation, on one hand provides a frequency-domain description of the waveform with convergence characteristics guaranteed by the specific adopted base, and on the other hand it provides the reconstruction of the state variables in time-domain within a switching interval. Depending on the analysis aspect that is needed, we may privilege one aspect or the other.

For example, for a control design purpose, by isolating the behavior related to the scaling function, it is possible to perform traditional small signal analysis for feedback control synthesis. By looking at all the wavelet coefficients in a steady-state analysis it is possible to optimize the size of the passive components, though working with a very limited number of coefficients in comparison with a Fourier-based analysis. And finally, looking at all the wavelet coefficients the impact of different modulation strategies can be quickly compared both in transient and steady-state operation.

In what follows, we consider a set of examples of power converters, that are analyzed via Haar transformation, according to what is stated above. Different wavelet transformations could be adopted, without changing the structure of the model. All the simulation results have been obtained by means of Matlab scripts, exploiting the fact that the wavelet transformation and the time-discretization turn the integro-differential problem in a matrix form problem.

3.1 The Case of the Buck Converter

We consider here the case of a buck DC-DC converter, whose simplified scheme is reported in Figure 3, operating with a 30% duty-cycle in Continuous Conduction Mode (CCM).

The analysis is performed with four coefficients and therefore four time-slots are identified within each switching period. The whole system is described with eight Haar coefficients, four for each of the two state variables.

The state equations for the two configurations are:

\[
\begin{bmatrix}
\dot{i}_L \\
\dot{v}_C
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\tau} \\
\frac{1}{\tau} & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix}
\dot{i}_L \\
\dot{v}_C
\end{bmatrix} + \begin{bmatrix}
\frac{1}{\tau} \\
0
\end{bmatrix} [V_{in}] \quad \text{for} \quad T_{on}
\]

\[
\begin{bmatrix}
\dot{i}_L \\
\dot{v}_C
\end{bmatrix} = \begin{bmatrix}
0 & -\frac{1}{\tau} \\
\frac{1}{\tau} & -\frac{1}{RC}
\end{bmatrix} \begin{bmatrix}
\dot{i}_L \\
\dot{v}_C
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} [V_{in}] \quad \text{for} \quad T_{off}. 
\]  

(20)

Since the buck converter operates in CCM with a 30% duty-cycle, we can assume that the configuration for \( T_{on} \) holds for 1/3 of the switching period and the configuration for \( T_{off} \) holds for the rest of the switching period, that is 2/3 of the switching period. The duty-cycle does not need...
According to what previously presented, the Haar representation of this system is:

\[ X_H(k+1) = [F_w] X_H(k) + [G_w] U_H(k) \]  

where \( X_H \) is an eight-element vector, comprising four coefficients for each of the two state variables. The parameters are set as follows: \( R = 5\Omega, L = 10\text{mH}, C = 20\mu\text{F}, \) switching period equal to 0.5 ms and input voltage is 30 V. Figure 4 shows the results during a transient at fixed duty-cycle, as obtained from the Haar analysis. Notice that the ripple, due to the switching, is clearly visible in the waveform obtained with the Haar analysis. In Figure 5 the evolution of the coefficients related to the scaling function is shown together with the state-space averaged variable.

It is to be highlighted how these two quantities perfectly match, for each of the state variables. This characteristic configures the averaged model as a particular case of the wavelet representation, that is, the wavelet model contains the averaged model in its coarser representation. We increased the detail of reconstruction of the real waveform by increasing the order of the wavelet decomposition.

3.2 The Case of the Boost Converter

We now consider the boost converter as a case study. The boost converter will be simulated both in CCM and Discontinuous Conduction Mode (DCM) operation. In the following sections we will refer to the basic configuration reported in Figure 6.

The boost converter in CCM operation allows two configurations: one during \( T_{on} \) when \( S_1 \) is in its ‘on’ state, the other during \( T_{off} \) when \( S_1 \) is in its ‘off’
Figure 5. Transient evolution of the averaged model compared with the wavelet component of the scaling function. Diagram a) for the voltage across the capacitor and b) for the current through the inductor.

Figure 6. Basic configuration of the boost converter.

3.3 The Boost Converter in Transient Conditions

The following parameters are used for the study of the boost converter in transient conditions: $R_L = 1\text{m} \Omega$, $R = 10\Omega$, $L = 0.1\text{ mH}$, $C = 1\text{ mF}$, $V_{in} = 20\text{ V}$, switching frequency $= 20\text{ kHz}$, duty-cycle $= 0.5$. The interesting issue of this example is that the steady-state operating condition is in CCM, but the system undergoes DCM conditions during the transient.

The computation is carried out with eight wavelet coefficients and again with 64 wavelet coefficients. The results are shown in Figure 7 and Figure 8 respectively.

The computation was performed with no a priori knowledge of the time occurrence of the DCM condition. The way the wavelet approach manages the DCM is explained in detail in Section 3.5. It is interesting here to point out that the steady-state condi-
tion was correctly identified, within the approximation allowed by the refinement of the representation, and in spite of the occurrence of DCM operation during transient.

In order to verify the results of the wavelet analysis, an analysis based on a conventional time-domain simulator has been performed. We adopted the VTB platform, developed at the University of South Carolina [34] [35].

Figure 9 illustrates the schematic of the converter under analysis in the VTB environment. The Conv0 element embeds the model of the boost converter power components.
The MOSFET and the diode are described as switching resistances.

Figure 10 illustrates the results of the VTB simulation. Both current (top part of the figure) and voltage (bottom part of the figure) show good agreement with the Haar domain simulation previously reported in Figure 8. This result is particularly significant considering the highly nonlinear nature of the transient behavior. Because of the fixed duty-cycle operation, the converter operation transitions from CCM to stocktickerDCM and then back to CCM. This is not a practical approach to the control of the converter, but it is significant for modeling purposes,
since it shows the capability of the Haar model to switch from one model formulation to another automatically.

Furthermore, it is interesting to compare the steady state of the simulation with the steady-state analysis performed in the Haar domain. As described in the next paragraph, the Haar model can catch this information directly and in closed-form, while with time-domain analysis we need to wait for the transient evolution.

### 3.4 The Steady-State Operating Condition

Let us now consider the problem of computing the steady-state condition for the converter operating at fixed duty-cycle. This problem has been studied in detail and different solutions are available in literature. While it is reasonably simple to calculate the steady-state averaged behavior, it is more complex to calculate the steady-state solution when the ripple is taken into account. Some methods proposed in the literature are based on iterative runs of simulation models [37] while other methods approach the analytical problem directly [36, 38, 39]. In particular the problem can be posed as harmonic balance or also wavelet balance [33]. Steady-state analysis methods, completely based on wavelet analysis, have been also proposed by Tam, Wong and Tse [40, 41]. Here we propose a solution based on the properties of the wavelet state-space formulation, showing that the approach introduced in the previous paragraphs can cover both transient and steady-state analysis.

The steady-state operation is defined in terms of wavelet coefficients as the condition that satisfies the following equality:

$$ X_w(k + 1) = X_w(k). $$

This condition states that the wavelet coefficients of the state variables are the same in two subsequent switching periods when the system is in steady-state. This definition of the steady-state condition carries information of the averaged value and of the ripple, with a detail limited only by the number of wavelet coefficients. Applying the steady-state condition to the wavelet representation of the system leads to the following expression:

$$ X_{wst}(k + 1) = ([I]_{n_w,N} [F_w] - [G_w]^{-1}) \times [G_w] [U_w] $$

where $n_w$ is the number of state-variables and $N$ is the chosen number of wavelet coefficients, coincident with the number of time slots.

The wavelet approach to the direct computation of the steady-state condition with Equation (24) was applied to the boost converter in Figure 6 with the same parameters of the circuit in the previous section. The computation of the steady-state condition, in the case of the boost converter, leads to the curves shown in Figures 11 and 12 represented over one switching period. The capacitor voltage waveform clearly shows the correct average value and the switching ripple due to the contribution of the current

---

**Figure 11.** Capacitor voltage during a switching period as obtained from the direct computation of the steady-state condition.
coming from the diode during the off phase. The steady state directly computed matches the steady state reached after the transient simulation.

3.5 The Boost Converter in DCM

Let us now take a closer look at the operation of the boost converter in DCM. For given values of the system parameters and of the duty cycle, DCM operation may occur. In such case, the system has a third configuration different from the two previously considered during $T_{on}$ and $T_{off}$ in CCM. The equation modeling the behavior in DCM is reported here:

$$\begin{align*}
\dot{i}_L &= 0 \\
\dot{v}_C &= -\frac{1}{RC} v_C
\end{align*}$$

To account for the possibility of DCM to occur, the computation algorithm that implements the wavelet model performs a check on the value of the inductor current. A negative current is interpreted as a transition to DCM during the last time slot of the $k$-th period under consideration. In such an event, the computation is repeated for the last period, assuming that the circuit configuration during the last time slot is that of DCM, as in Equation (25). Thus, having defined $F_1$ and $G_1$ and $F_2$ and $G_2$ as the matrices of the two CCM configurations, we have to introduce $F_3$ and $G_3$ as the matrices describing the configuration during DCM. The calculation is therefore repeated assuming the system in configuration three during the last time slot. The value of the inductor current is then checked again. In case of negative value we assume the DCM occurred starting from the penultimate time slot, so the computation is iterated as previously described. This process is repeated until the DCM transition interval is found.

To allow for the analysis of a case of DCM operation in steady-state condition, the parameters of the boost converter circuit are reset. In particular: $R_l = 2\Omega$, $L = 10\, \mu H$, $C = 100\, \mu F$, $R_{out} = 30\, \Omega$. The duty-cycle is fixed to 0.125, while the system is still modeled with eight wavelet coefficients.

The algorithm used for the computation performs the check on the inductor current sign as previously described, to indentify the occurrence of DCM. The behavior of the inductor current and the capacitor voltage are reported in Figures 13 and 14 respectively. In Figure 13, only the Haar scaling function and mother wavelet coefficients are used.

The side result of the computation of the transient under DCM condition is the synthesis of the matrix carrying the information about the DCM occurrence. In fact, the iterations previously described to identify the occurrence of DCM, lead to a transition matrix $[F_w]$ that contains a good approximation of the DCM behaviour. The representation can be as good as needed, provided that a large enough number of wavelet coefficients is adopted. Thanks to the transition matrix thus obtained, the steady-state solution for different inputs can be computed directly. This ability of the model to self-reconfigure when DCM occurs, allows the convenient simulation of a complicated transient as the one previously reported in Figure 7.
Figure 13. The inductor steady-state current during DCM operation with duty-cycle equal to 0.125

Figure 14. The capacitor steady-state voltage during DCM operating condition with duty-cycle equal to 0.125

3.6 Steady-State Resonant Converter

A more challenging steady-state analysis case is provided by the study of a Zero Current Switching resonant converter. An example of such topology is illustrated in Figure 15. The state equations for this circuit can be written as:

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Figure 15. A ZCS topology

Figure 16. Steady-state operation of the ZCS converter with six orders of wavelet

\[
\begin{bmatrix}
    i_s \\
    \dot{v}_{cr} \\
    i_{lf} \\
    \dot{v}_{cf}
\end{bmatrix} = \begin{bmatrix}
    -\frac{1}{L_f} & 0 & 0 & 0 \\
    0 & 0 & -\frac{1}{R_s L_f} & 0 \\
    0 & -\frac{1}{C_f} & 0 & -\frac{1}{R_f C_f} \\
    -\frac{1}{C_r} & 0 & \frac{1}{C_f} & -\frac{1}{R_D C_r}
\end{bmatrix} \begin{bmatrix}
    i_s \\
    v_{cr} \\
    i_{lf} \\
    v_{cf}
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    \frac{1}{R_f C_f}
\end{bmatrix} [V_{dc}].
\]

Where

- \(i_s\) is the current through the main switch
- \(v_{cr}\) the voltage across the resonant capacitor
- \(i_{lf}\) is the current through the filter inductance
- \(v_{cf}\) the voltage across the filter capacitor
- \(R_s\) and \(R_D\) are respectively the resistance of the main switch and of the diode.

These resistances can switch between two values, representing respectively the on and the off condition. As a result, there are four different possible configurations for each period of operation. The same iterative procedure proposed for the boost converter can be here applied to identify the four different stages of operations in steady-state. Figure 16 shows the results obtained by using six orders of wavelet.
orders of wavelet. Such a solution is obtained in a very limited time running a Matlab script and the results are consistent with the theoretical behavior of such a topology as reported for example in Mohan et al. [43].

Figure 17 shows the same results but with eight orders of wavelet. While the accuracy is improved by developing the order of the model, it is interesting to notice that also at lower resolution it was possible to obtain a quite accurate reconstruction of the steady-state behavior with limited computational effort.

3.7 The Case of DC-AC Converters

The wavelet approach can also be applied to DC-AC converters. We here reported the results of the analysis of a DC-AC converter for contactless energy transfer, as reported by Monti and Ponci [42]. The reference topology, with a single-phase output, is shown in Figure 18. The load on AC side is a transformer operating at the switching frequency feeding a resistive heating unit.

Figure 19 illustrates the equivalent circuit of the load. The combination of $R$, $L_{dt}$ and $L_m$ represents a simple model of the transformer, $R_{load}$ a resistive load for heating purpose. The capacitance $C$ is inserted to optimize the energy transfer compensating for the inductive effect of the transformer. The sizing of such capacitance is one of the key issues of the design of this system.

In particular let us assume that the converter is operating in full wave mode (square wave at the switching frequency of 200 kHz). Under such conditions, the Haar wavelet representation is extremely compact: only the mother wavelet coefficient will be different from zero fully representing the input waveform.

The calculation can be easily repeated for many different values of the capacitance searching for the optimal result under highly distorted conditions. Figure 20 shows the load current calculated with steady-state wavelet analysis, under the assumption that the capacitor completely compensates for the inductive effect at the switching frequency.

The wavelet approach to the modeling of single-phase DC-AC converters can be extended to three-phase con-

![Figure 17. Steady-state operation of the ZCS converter with eight orders of wavelet](image)

![Figure 18. Basic circuit of a DC-AC single-phase converter](image)
Let us consider the Park model of a three-phase inverter. The Park model can be described by the equivalent circuit in Figure 21. The following equations hold:

\[
\begin{align*}
\frac{di_l}{dt} &= -\frac{R}{L}i_l - \frac{v_c}{L} + \frac{V_{dc}}{L} \\
\frac{dv_c}{dt} &= \frac{i_l}{C} - \frac{f_d}{C}i_d - \frac{f_q}{C}i_q \\
\frac{di_d}{dt} &= \frac{f_d}{L}v_c - \frac{R}{L}i_d \\
\frac{di_q}{dt} &= \frac{f_q}{L}v_c - \frac{R}{L}i_q
\end{align*}
\]

where \(f_d\) and \(f_q\) are the \(d\)-axis and \(q\)-axis modulation indexes, \(i_d\) is the AC side Park direct axis current component, \(i_q\) is the AC side Park quadrature axis current component, \(R\) and \(L\) are the resistance and inductance of the AC load, \(i_l\) is the DC side inductor current, and \(v_c\) is the DC side capacitor voltage.

The Park transformation effectively turns the three-phase DC-AC converter into an equivalent two-output DC-DC converter, as shown in Figure 21. As a consequence, the wavelet approach presented for DC converter can immediately be applied. The wavelet representation of the state variables, also in this case, provides information not only about the averaged value but also about the ripple.

The voltage across the capacitor and the current of the inductor, the chosen state variables, are computed with the proposed method and the results are reported in Figure 22. The corresponding load current in the three-phase reference frame is reported in Figure 23.

The computation of the three-phase steady-state load current is carried out according to the approach presented for DC-DC converters. The three-phase system in fact, was reconducted to the DC-DC case with the Park transformation, and therefore the direct computation of the steady-state condition in the Haar domain can be applied.

The steady-state results thus obtained are presented in Figure 24.

### 4. The Haar Eigenvalues

The availability of a state-space model describing the dynamic behavior of a switching power converter also opens the door for the analysis of the eigenvalues of the new state matrix. This analysis, when combined with a model of the closed loop feedback control, can provide important stability analysis information. Also working on the open loop model, we can acquire useful information about the dynamic of the converter while varying significant parameters of the circuit. Notice that, because the wavelet transformation can be considered a similarity transformation, the analysis of the eigenvalues brings the same information as in time domain.

Let us study, for example, how the small-signal eigenvalues of the system are affected by the variation of the
duty-cycle. The steady state is computed in the Haar domain for each value of this parameter as previously described. This computation accounts for a possible DCM, so the wavelet transition matrix $F_w$ in steady state eventually carries the trace on the possible DCM. The $F_w$ matrix relates the wavelet coefficients of the state variables in two subsequent periods. Eigenvalues can be extracted from this $F_w$. The eigenvalues thus extracted come in multiple sets: a number of sets equal to the number of wavelet coefficients used in the model and a number of eigenvalues in each set equal to the number of state variables.

Let us examine the case of the boost converter, starting with the system operating in CCM. In such a case the standard eigenvalues of the system can be computed in closed form according to the following formula:

\[
\lambda_{1,2} = -\left(\frac{R}{2C} + \frac{1}{\text{out}}\right) \pm \sqrt{\left(\frac{R}{2C} + \frac{1}{\text{out}}\right)^2 - \frac{4R}{L}\frac{\text{out}}{C}}.
\] (28)

The closed form results computed with Equation (28) are then compared to eigenvalues of the wavelet transition matrix. The duty-cycle spans the range from zero to one, and the other parameters are: $R_1 = 2\Omega$, $L = 10\mu H$, $C = 100\mu F$, and $R_{\text{out}} = 3\Omega$.

The locus of the eigenvalues of this second-order boost converter, are mapped in the Z-domain with the duty-cycle as a parameter changing from zero to one and the two-branch locus shown in Figure 25. The eigenvalue locus computed with the closed form approach is superimposed to the locus obtained from the wavelet approach. This result is obtained with eight wavelet coefficients.
Let us now change the value of the inductance to $L = 5\mu\text{H}$, so that the DCM operation actually occurs for given values of the duty-cycle. The corresponding results are reported in Figure 26. On the right side of the figure it is possible to observe a change in the slope of the locus branches that corresponds to the transition from DCM to CCM.

The computation was performed under the same conditions as the CCM case. The iteration identified the transition to DCM and produced the wavelet transition matrix that accounts for the DCM configuration.
Figure 25. Locus of the eigenvalues of the boost converter in CCM, as computed in closed form and with the wavelet approach, the two branches as computed with the two methods are virtually indistinguishable in the figure.

Figure 26. Locus of the eigenvalues of the boost converter in DCM, as computed in closed form and with the wavelet approach.

5. Conclusions

This paper presents the development and application of wavelet transform for the modeling and analysis of power converters. As sample applications, some basic converters, both DC-DC and AC-DC, are considered. The proposed approach yields a compact representation and allows for the analytical calculation of the steady-state condition of a switching circuit operating at constant frequency. Both CCM and DCM were considered, showing that the wavelet models can easily detect the transition from one mode of operation to the other. The outcome
of this iterative procedure is the determination of a comprehensive matrix representation that carries the DCM information. Finally, the wavelet approach was applied to three phase converters, to demonstrate that such extension is feasible.

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7. References


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