

# Modeling Power Diode by Combining the Behavioral and the Physical Models

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**Abstract** - This paper presents an approach of implementing the dynamic multi-resolution power diode model in the Virtual Test Bed (VTB), a platform for complex system simulation and prototyping. The new model is a combination of the behavioral model and the physical model. The behavioral model is based upon the widely used Shockley equation; its advantages are simplicity and fast simulation speed, however it is incapable of representing the forward and reverse recovery phenomena during the state switch, which is required in many applications. The physical model is based upon the lumped charge theory; it has higher fidelity than the behavioral model as well as the ability of representing the forward and reverse recovery. The physical model's drawback is its complex mathematical representation that also requires a sub-micro-second time step, which makes the simulation extremely slow. The combined model inherits the advantages of both models; the validation example shows the simulation speed can be doubled while the accuracy loss is less than three percent.

The new model has two resolution levels: the behavioral level and the physical level. The physical level is applied only when the diode changes state. The new model inherits the advantages of higher fidelity and faster speed from both of its parents. The validation example shows that the execution time can be reduced by half, while the accuracy loss is less than three percent.

The new model is implemented in the Virtual Test Bed (VTB) environment, which is a topology-oriented time-domain simulator that employs the Resistive Companion Form (RCF) method as the primary simulator. In the following sections, the behavioral and the physical models, along with their representations in the RCF method, are presented first. Next, the approach of combining the two models into the multi-resolution model is shown. Finally, a validation example is given to prove the efficiency of this approach.

## I. INTRODUCTION

Modeling the transient behavior of power diodes, such as the forward and reverse recovery, is very important to many power electronic circuit simulations, especially to the simulations of switching power loss, conducted EMI and any other waveform sensitive effects.

Existing power diode models can be classified as either behavioral models or physical models. The behavioral models are normally simpler and are not directly related to the internal physical processes in the device, such as the widely used Shockley model. They are easier to implement and cause relatively low computation load in simulation; however, most of them are incapable of correctly representing the diode behaviors during the brief transitions between the forward and reverse conduction modes, such as the forward and reverse recovery phenomena, which are tightly related to the internal physical processes. In contrast, most physical models include the equations for drift and diffusion of electrons and holes, which enable the models to represent the transient behaviors precisely. However, the physical models usually contain many mathematical equations and parameters, which are not only complicated to implement but also cause severe computation load during simulation.

In this paper, we present an approach of combining the behavioral model and the physical model into a multi-resolution power diode model and dynamically switching between these models at appropriate times during simulation.

## II. MODEL DESCRIPTION

### A. The modified Shockley Model

The Shockley model is widely used to describe the steady-state voltage-current relationship for a p-n junction diode. It can be described as,

$$i_{p-n} = I_s \left[ \exp\left(\frac{v_{p-n}}{\eta V_T}\right) - 1 \right] \quad (1)$$

The junction current and voltage are represented by  $i_{p-n}$  and  $v_{p-n}$  respectively,  $\eta$  is the ideality factor,  $I_s$  is the saturation current, and  $V_T$  is the thermal voltage of the diode, which is defined by

$$V_T = \frac{kT}{q} \quad (2)$$

Where  $k$  is Boltzmann's constant ( $1.38 \times 10^{-23}$  J/K),  $q$  is the protonic charge ( $1.6 \times 10^{-19}$  C), and  $T$  is the Kelvin temperature. At room temperature (20-25°C),  $V_T$  is approximately 25 to 26 mV.

Considering that the physical model should also include the effect of the series resistance, the Shockley model is modified as

$$v_D = v_R + v_{p-n} = i_D R_s + \eta V_T \times \ln\left(\frac{i_D}{I_s} + 1\right) \quad (3)$$

The next step is to transform (3) into the Resistive Companion Form. The RCF modeling method employs the solution of the discretized form of the time differential equations that can be represented as the solution of a simple DC circuit at any instant of time [7]. The diode has two terminals through which it can be interconnected to other components. Each terminal has an associated across and through variables. Since this component is electrical, these variables are the terminal voltage, with respect to a common reference and the electrical current flowing into the terminal, respectively. The diode's RCF equivalent DC circuit is represented as Fig. 1.

Assuming the simulation time step is  $h$ , let  $v = V_1 - V_2$ ,  $i = I_1 = -I_2$ , by discretizing the modified Shockley equation (3) with the trapezoidal algorithm, which is the most commonly used algorithm in the VTB, we can get

$$v(t) = v(t-h) + \frac{dv}{di}\bigg|_{i(t-h)} \times [i(t) - i(t-h)] \quad (4)$$

$$= v(t-h) + \left[ R_s + \frac{\eta V_T}{i(t-h) + I_s} \right] \times [i(t) - i(t-h)]$$

Rearranging,

$$i(t) = \frac{i(t-h) + I_s}{R_s i(t-h) + R_s I_s + \eta V_T} \times v(t) - \frac{v(t-h)i(t-h) + v(t-h)I_s - R_s i(t-h)^2 - R_s I_s i(t-h) - \eta V_T i(t-h)}{R_s i(t-h) + R_s I_s + \eta V_T} \quad (5)$$

Let  $\vec{I} = [i_1(t) \ i_2(t)]^T$ ,  $\vec{V} = [v_1(t) \ v_2(t)]^T$ , the RCF model can be rewritten as  $\vec{I} = G \cdot \vec{V} - \vec{B}$ , where the Jacobian matrix  $G = \begin{bmatrix} g(t-h) & 0 \\ 0 & -g(t-h) \end{bmatrix}$ , and the initial state vector  $\vec{B} = [b(t-h), \ -b(t-h)]^T$ .

### B. The Lumped Charge Model

The physical model applied here is an extension of the basic charge-controlled diode model using the lumped charge concept of Linvill [8]. The extended model includes the equations for both forward and reverse recovery, as well as emitter recombination [1]. This model acts precisely for the whole range of circuit operating conditions. However, it contains nine mathematical equations, including a piecewise nonlinear differential equation, which makes the model very difficult to implement and time-consuming to solve repeatedly in simulation.

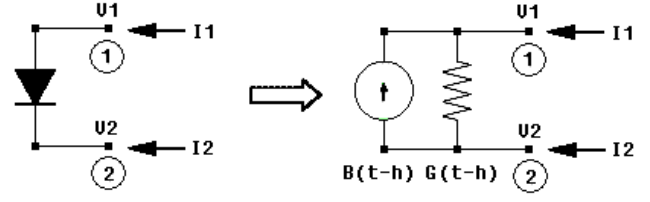


Fig. 1. Resistive Companion Form model

For typical device parameters, convergence of the mathematical representation also requires a sub-microsecond time step, which causes severe computation load and makes the simulation extremely slow. Nonetheless, the model, which is described next, can be used effectively during the brief transitions between forward and reverse conduction modes.

The diode current and voltage equations, including the effects of the series resistance and the junction capacitance, are

$$i = i_E + i_M + 2C_j \times dv_E / dt \quad (6)$$

when  $v_E \geq 0.25\phi_b$ ,

$$C_j = C_{j0} \times (1 - 2v_E / \phi_b)^{-0.5} \quad (6.1)$$

when  $v_E < 0.25\phi_b$ ,

$$C_j = \sqrt{2} \times C_{j0} \times 2v_E / \phi_b + C_{j0} / \sqrt{2} \quad (6.2)$$

$$v = 2v_E + 2v_M + R_s \times i \quad (7)$$

The reverse recovery related equations are,

$$i_M = (q_E - q_M) / T_M \quad (8)$$

$$(q_E - q_M) / T_M = dq_M / dt + q_M / \tau \quad (9)$$

$$q_E = I_s \times \tau \times [\exp(v_E / V_T) - 1] \quad (10)$$

The forward recovery related equation is,

$$v_M = V_T \times T_M \times R_{M0} \times i / (q_M \times R_{M0} + V_T \times T_M) \quad (11)$$

The emitter recombination related equation is,

$$i_E = I_{SE} \times [\exp(2v_E / V_T) - 1] \quad (12)$$

Where

$q_E$  - charge at the junction boundary

$q_M$  - charge at the i-region

$v_M$  - voltage across half of the i-region

$v_E$  - voltage across the junction boundary

$i_E$  - emitter region recombination current

$i_M$  - diffusion current

$T_M$  - transit time

$\tau$  - carrier lifetime

$I_s$  - saturation current

$I_{SE}$  - emitter region saturation current  
 $R_{M0}$  - initial resistance of the i-region  
 $R_s$  - series resistance  
 $V_T$  - thermal voltage  
 $C_j$  - junction capacitance  
 $C_{j0}$  - zero-biased junction capacitance  
 $\phi_b$  - zero biased built-in potential

Since the deduction and the detailed RCF representation of the physical model are too complicated to present here, we use a simplified form (13) to help with the explanation. Using the modified nodal analysis method, seven internal virtual nodes are required in the RCF representation, along with six internal state variables,  $i_E$ ,  $i_M$ ,  $v_E$ ,  $v_M$ ,  $q_E$  and  $q_M$ . The directions of  $v(t)$  and  $i(t)$  are the same as in Fig. 1.

$$\begin{bmatrix} i_1(t) & i_2(t) & \frac{0 \dots 0}{\gamma} \end{bmatrix}^T = [G(t-h)]_{(9 \times 9)} \cdot \begin{bmatrix} v_1(t) & v_2(t) & i(t) & i_E(t) & i_M(t) & v_E(t) & v_M(t) & q_E(t) & q_M(t) \end{bmatrix}^T - \bar{B}(t-h) \quad (13)$$

### III. COMBINING THE BEHAVIORAL MODEL AND THE PHYSICAL MODEL

#### A. The effective range of the physical model

Except for the ideality factor  $\eta$ , the other two parameters of the Shockley behavioral model, the saturation current  $I_s$  and the thermal voltage  $V_T$ , are a subset of the parameters of the physical model. If the two parameters are the same as the physical model, we can find that the outputs of both models match very well, except for a small range during the state switch, as in Fig. 2.

As stated before, the physical model will slow the simulation significantly due to its complex representation and small time step, so its effective range should be limited within the state switch period. Simplistically, the state switch occurs at the instant of voltage or current zero-crossing. Noticing that the current of the behavioral model does not pass zero, a more appropriate word is "zero-closing". However, since the discrepancy between the two models becomes larger and larger before the state switch, as shown in Fig. 3, which may incur significant error in the initialization of the physical model, it is necessary to apply the physical model before the actual state switch.

The value of current is used to determine the start point of the reverse recovery, which happens when a diode is shutting off. To minimize the effect of the increasing discrepancy, the physical model is applied once the current drops into a preset threshold value. In a similar way, the voltage decides the

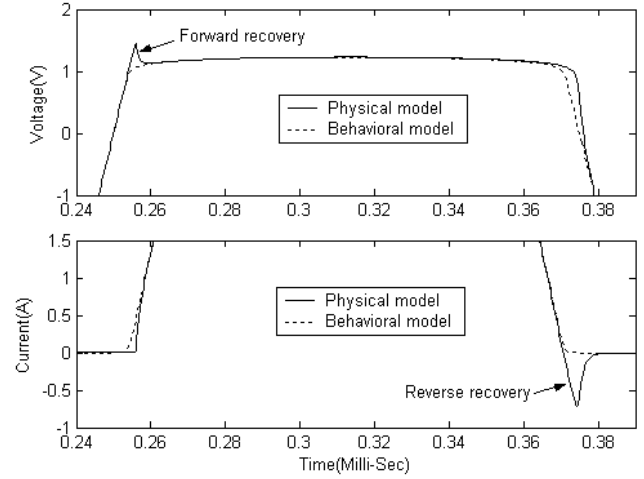


Fig. 2. Comparison between the behavioral and the physical model

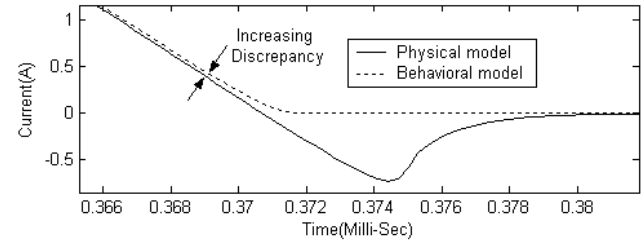


Fig. 3. Increasing error before the state switch

start point of the forward recovery, which happens when the diode is turning on.

To expedite the simulation, the behavioral model should resume the control right after the state switch is completed. The scenarios of the reverse and the forward recovery are considered separately. Noticing that a power diode operates at high voltages, the effect of the i-region must be considered. From the lumped charge theory, the simplified voltage and charge distribution in a p-i-n junction can be shown in Fig. 4.

The reverse recovery is caused by the charge  $q_M$  that is stored in the i-region when the diode is conducting. The recovery can be considered completed when  $q_M$  is depleted to zero or a very small value. As for the forward recovery, it is caused by the initial low conductivity in the i-region; the total voltage across the i-region  $2v_M$  will decrease quickly to the normal steady state junction forward drop, as the injected carrier concentration increases. Therefore, the completion of the forward recovery can be determined by comparing  $2v_M$  to the junction forward drop voltage. Fig. 5 shows the curves of  $q_M$  and  $v_M$  in correspondence with  $v$  and  $i$ .

#### B. Initialization of the physical model

The initialization of the behavioral model is not a problem because it does not have any internal state variables; the initial values of the diode voltage and current can be obtained

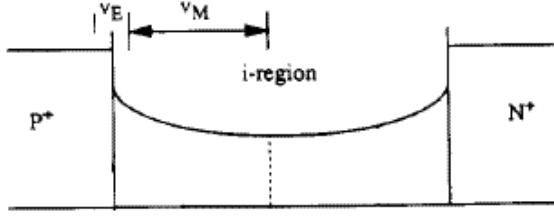


Fig. 4. Voltage and charge distribution in a p-i-n junction

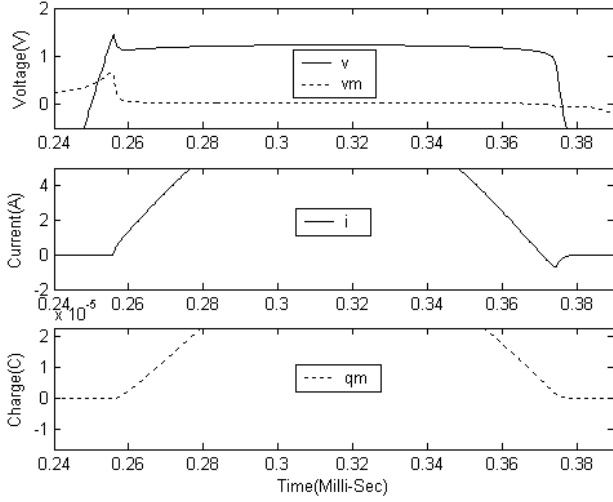


Fig. 5.  $v_M$  and  $q_M$

directly from the physical model. However, the initialization of the physical model is much more complex. The physical model contains six internal state variables that need to be initialized. Among them  $q_E$  and  $i_E$  can be obtained directly from  $v_E$  by (10) and (12) respectively, so the independent variables are  $v_E$ ,  $v_M$ ,  $q_M$ ,  $i_M$ . Considering (6) is a piecewise nonlinear differential equation, it is very difficult to get the exact solutions by solving these equations with the initial values of  $v$  and  $i$  obtained from the behavioral model; some approximations must be made. Observing  $i_E$  and  $i_M$  as the dominant components of the diode current, the effect of the junction capacitance is neglected during the initialization. As shown in Fig. 6, the diode current and the summation of  $i_E$  and  $i_M$  are very close under normal operating conditions.

Then (6) can be simplified as,

$$i = i_E + i_M \quad (14)$$

Substitute  $q_E$  from (10) into (8), considering  $i_M = i - i_E$  and (12), then  $q_M$  can be obtained. Apply it into (11), since  $v_M$  can be represented by  $v_E$  from (7), we can get the equation to solve  $v_E$  as,

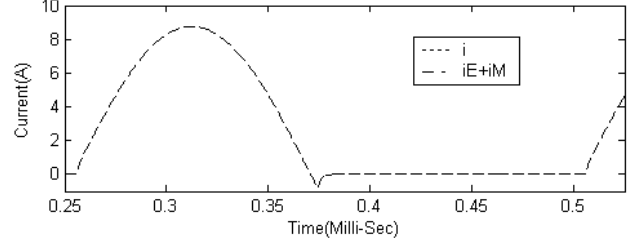


Fig. 6.  $i$  and  $i_E + i_M$

$$\frac{v - R_S \times i}{2} - v_E = \frac{V_T \times T_M \times R_{M0} \times i}{R_{M0} \times \left\{ I_S \times \tau \times \left[ \exp\left(\frac{v_E}{V_T}\right) - 1 \right] - i \times T_M \right\} + V_T \times T_M} \quad (15)$$

Noticing  $v$  and  $i$  are constants here,  $v_E$  is the only unknown variable to be solved. Since it is very difficult to employ the analytical method, we choose the numerical iterative method instead. Rearranging (15), let

$$\begin{aligned} f(v_E) = & R_{M0} \times I_S \times \tau \times \exp\left(\frac{v_E}{V_T}\right) \times \left(\frac{v - R_S \times i}{2} - v_E\right) \\ & + v_E \times (R_{M0} \times I_S \times \tau + R_{M0} \times i \times T_M - V_T \times T_M) \\ & + \frac{v - R_S \times i}{2} (V_T \times T_M - R_{M0} \times I_S \times \tau - R_{M0} \times i \times T_M) \\ & - R_{M0} \times V_T \times T_M \times i \end{aligned} \quad (16)$$

The Newton-Raphson method described in (17) is employed to solve the  $v_E$  that makes  $f(v_E) = 0$ .

$$v_{E,k+1} = v_{E,k} - \frac{f(v_{E,k})}{f'(v_{E,k})} \quad (17)$$

$v_M$ ,  $q_E$ ,  $q_M$ ,  $i_E$  and  $i_M$  can be obtained accordingly from  $v_E$ . By now the physical model has been fully initialized from the behavioral model.

#### IV. MODEL VALIDATION

Fig. 7 is the validation test bench. The voltage source is an adjustable high-frequency sinusoidal source; the load is idealized as a resistor and the diode is in series with a small inductor. The model-switching related parameters are,

Switching voltage threshold:  $-0.3$  Volt;  
Switching current threshold:  $0.45$  Ampere;

Switching i-region charge  $q_M$  threshold: 1.0E-8 C;  
 Junction forward drop voltage (also a parameter of the physical model): 0.6 Volt.

Fig. 8 depicts the comparison between the simulation results of the dynamic model and the physical model; the difference can hardly be distinguished. When the model is switched to the behavioral level from the physical level, the simulation time step will also be amplified by a factor that is designated by the user. Since the behavioral level is applied in the steady state, the factor does not affect the fidelity much. However, it does affect the simulation speed; a larger factor will incur faster speed. Considering the stability, we choose five as the time step factor in this application.

TABLE 1 compares the execution times of different simulation modes. By working in the dynamic mode, the execution time can be reduced by 48%. As for fidelity, compared to the outputs of the physical model, the average relative error of the dynamic model is less than three percent.

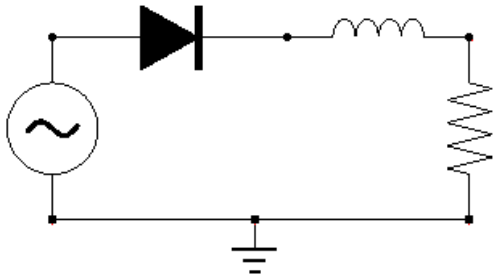


Fig. 7. Validation test bench

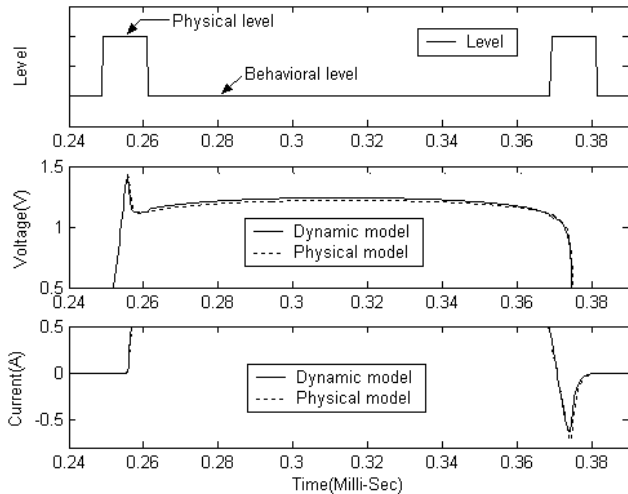


Fig. 8. Comparison between the dynamic and the physical model

TABLE 1  
 EXECUTION TIME OF DIFFERENT SIMULATION MODES

Mode	Applied Level	Time Step ( $\mu$ Sec)	Execution Time (Sec)
Fixed	Physical	0.3	36.4
Dynamic	Behavioral / Physical	1.5 / 0.3	19.1

## V. CONCLUSION

We have presented a method of combining the steady-state Shockley behavioral model and the lumped-charge physical model into a dynamic multi-resolution power diode model. This method has been successfully implemented in the VTB environment. The diode voltage, current along with the i-region charge and voltage are used to determine the effective range of the physical model. The Newton-Raphson method is employed to solve the initial conditions of the physical model. The validation example shows that the dynamic model can reduce the execution time by half with only 3% accuracy loss. This approach is not limited to the VTB; it can be applied in other circuit simulators as well.

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