Application of Quantized Discrete Event Simulation Methods to Naturally Coupled Systems

Charikleia Mamai, Anton H.C Smith, Igor Kondratiev, Roger A. Dougal
Dept of Electric Engineering, University of South Carolina–Columbia, USA

(mamai@email.sc.edu, antons@email.sc.edu, kondrati@email.sc.edu, dougal@cec.sc.edu)

Index Terms – Quantized State Systems, Discrete Event Simulation, Natural Coupling Methods

Abstract

A method for applying quantized discrete event simulation (DEVS) methods to naturally coupled system models is defined. Pairing the DEVS formalism with the Modified Nodal Analysis technique allows for asynchronous discretization of events, instead of the typical uniform time discretization while automatically enforcing natural conservation laws, unlike signal flow methods that propagate the output of one model to the next without adding physical constraint equations. The mathematical equations for naturally coupled DEVS (NCDEVS) are presented along with example formulations for inductor and capacitor circuit elements. Results from application of the technique to an RLC circuit and to an LC filtered Half-Wave rectifier driving a Permanent Magnet DC Motor (PMDC) are presented and shown to be close to the well known analytical solutions. A significant benefit of the NCDEVS formulation is that it allows bi-directional power flow, as shown in the case of the PMDC motor, therefore the solver doesn't need to be aware if the PMDC motor acts as a motor or as a generator. Moreover using NCDEVS, the motor is modeled by one set of equations, regardless of the operating mode, in contrast to the signal flow method, where the solver needs to have two separate models, one for each case.

1. INTRODUCTION

Discrete Event Simulation formalism (DEVS) was initially introduced by Zeigler in 1976[1] but the concept of Quantized Systems was formally defined after many years from the same author in [2]. Kofman et al. in [3-5] improved the original quantized state approach of Zeigler's, solving the issue of illegitimate models, and thus, showed that a discrete event driven simulation promises a significant improvement of computational efficiency in simulations of large and complex systems like power systems. However, known implementations of the DEVS formalism [6, 7] use the signal flow approach that requires manual insertion of the energy conservation laws at the coupling nodes, thus causing interoperability constraints. On the contrary, Modified Nodal Analysis (MNA) [8] that is widely used in commercial products such as PSpice, enforces the laws of conservation of energy and provides a simplified way for computer automated solution since a system model is created from models of individual components. When discretizing differential equations for use with MNA, a constant time-step is used which could be far smaller than necessary, especially as the system approaches steady-state. Usually, MNA uses time discretization that results in a huge computation overhead. Combining DEVS formalism with the concept of nodal analysis can remove any interoperability constraints and can furthermore reduce the computational overhead without sacrificing the accuracy of the simulation.

This work represents a first attempt to apply Quantized Discrete Event methods to the simulation of naturally-coupled systems. In a typical MNA system, when the differential equations are linearized and discretized in time with constant time step, h, the solution at a next time t_{n+1} can be found in terms of the state at time t by solving the Resistive Companion Equation (Eq. 1) [9-11]:

$$I(t+h) = G(t) \times V(t+h) - b(t) \tag{1}$$

Here, G (t) is the conductance matrix of the system. The n, m elements of G (t) are the total conductances attached between nodes n and m. Since G (t) can be trivially formed by summing the conductances of each element, these conductance values are referred to as being "stamped" into the conductance matrix, with each component contributed some conductance to each of the matrix elements to which it is attached.

For linear time invariant systems, the conductance matrix G (t) needs to be inverted only once at the beginning of the time stepping sequence and then the state trajectory of the system can be calculated afterwards simply by incrementing from one time step to the next. But for nonlinear or time varying systems, the conductance matrix must be inverted at each time step, which is somewhat costly in time. Temporal simulation of such nonlinear and/or time varying systems entails two main steps:

1. Computing the current through each component, in terms of the node voltages.

2. Computing the next node voltage, in terms of the component currents.

Each of these two steps can be performed in a variety of ways. In the work we present here, we solve step 1 by using a quantized DEVS approach. These current injections are then used in a conventional nodal solver to find the next value of node voltages, except that the next time of matrix inversion doesn't necessarily need to be uniformly stepped.

The Naturally Coupled DEVS (NCDEVS) method presented here was initially based on Nutaro's paper where he presents a discrete event simulation approach of a solution of a first order differential equation [12]. By considering the basic equation that describes an ordinary differential equation, taking the Taylor series expansion and neglecting the higher terms, it was possible to evaluate the change in time h that is required for a change of size of D, which is the change of state and our resolution. Since the Resistive Companion Equations (RCM) requires that the dynamic components should be expressed in the RCM equations and the equivalent circuits can be approximated, when the differential equation that describes the component and the integration method are known, we were able to proceed deriving the equivalent NCDEVS equations for a capacitor and an inductor. The resulting mathematical equations gave us the appropriate information on how the NCDEVS models of those dynamic components should look like. In our case, the inductor is stamped as controlled current source and the capacitor is stamped as a controlled voltage source.

Similarly to the classical RCM that solves for the differential equation that describes the dynamic component, the NCDEVS dynamic components formularization follows a similar approach. The dynamic component's differential equation is solved by using the DEVS discretization approach shown in [12]. The NCDEVS solver uses the MNA equations to describe the components, while using DEVS as a discretization method. In order to prove the validity of NCDEVS, we performed a simulation of an RLC circuit and a permanent magnet DC motor (PMDC). The simulation results of those circuits are presented in chapter 4 and they are compared to exact solutions of the corresponding system results. The outcome is proven to be close to the well known analytical results, with acceptable error variations.

Using a quantized DEVS approach, improves the performance of natural-coupled solutions by allowing for the time-step to increase as the rate of change in a system decreases. This is especially advantageous in dc power systems. Once the system reaches steady-state, stepping of the system components becomes extremely sparse in time. The total expected speedup of the system simulation should then be equal to $h_{avg} / h_{\_original.}$

2. NCDEVS

The purpose of this part of the paper is to present the background theory of the two methods that are behind the idea of NCDEVS. Therefore, we firstly present the Modified Nodal Analysis and secondly the DEVS formalization.

2.1 Modified Nodal Analysis

MNA provides a framework for creating a system description from devices that individually contain only the model for their own behavior; i.e. individual devices do not require knowledge of the other devices or the topology of the overall system. The model for each device is described using the Resistive Companion Method (RCM). The device can be considered as a "black-box" with a number of *terminals*, which are used to provide connections to other devices and each terminal is associated with one *across* and one *through* variable (current and voltage in the case of an electrical terminal). Then, the device is connected to neighboring devices via these terminals.

A set of terminals connected together form a node. Terminals of any physical type are supported, provided that those terminals satisfy energy conservation equations in the form of across and through variables associated with each terminal and the conductance matrix is what enforces those conservation equations. A major advantage of the resistive companion method over state-based solvers is that once the device models are constructed, any set of such interconnected devices can be easily assembled into a system Therefore, RCM is a technique which is appropriate for large system studies using an object oriented implementation.

The RCM requires that the dynamic components are expressed in the RCM equations and equivalent circuits. If the differential equation that describes the component is known as well as the integration method, the result is an algebraic equation that can be expressed by equation (1).

In equation (1), I (t + h) is the current at the terminal of the dynamic component at next time step, V (t + h) is the voltage across the component at next time step, G (t) is a coefficient with conductance unit and b (t) is an equivalent current representing the past history of the component. The relations between G (t), b (t) and the time step h that are the parameters of the device, depend on the integration method selected for the process. Equation (1) can be expressed in an equivalent circuit as Figure 1.

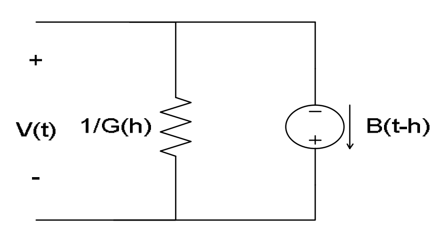


Figure 1. Resistive Companion equivalent model

Figure 1, shows that the Resistive Companion equivalent model looks a lot like a real current source with a current of b (t) and an internal resistance $G(t)^{-1}$.Therefore we can use the stamping method to express the Resistive Companion Model as shown in Table 1.

Component	A	x	b
RCM	$\begin{bmatrix} G(t) & -G(t) \\ -G(t) & G(t) \end{bmatrix} \begin{matrix} i \\ j \end{matrix}$ i j	$\begin{bmatrix} x1 \\ x2 \end{bmatrix}$	$\begin{bmatrix} b(t) \\ -b(t) \end{bmatrix}$

Table 1

By combining the differential equations and an integration method like Euler's, Trapezoidal etc, we can get the Resistive Companion equation for the components that we need to analyze. Each one of the integration methods present different advantages [13], but they share the common characteristic that they discretize the time. In NCDEVS, we are discretizing the state, rather than time, and we are solving for our next time h corresponding to the next change in the state of our system.

2.2 DEVS Formalization

The Quantized State Systems (QSS) showed that a generic system of ordinary differential equations can be modified with the addition of Quantization Functions, transforming the system into a QSS, whose behavior can be exactly represented by a DEVS model.

A continuous system represented by the state space description is shown in equation (2):

$$\dot{x}(t) = f(x(t), u(t), t) \tag{2}$$

where x (t) is the state vector and u (t) is the input vector. The corresponding quantized state system has the following form of equation (3):

$$\dot{x}(t) = f(q(t), u(t), t) \tag{3}$$

where q(t) is the quantized version of the original state vector x(t). For a quantization function the sgn or floor functions can be used.

2.3 NCDEVS of a First Order System

In NCDEVS we exploit the DEVS solution for first order differential equations since the use of first order differential equations in modeling and simulating process are vital in tools such as state-space modeling.

Consider an ordinary differential equation that can be written in the form of equation (4):

$$\dot{x}(t) = f(x(t)) \tag{4}$$

The Taylor series expansion of $\dot{x}(t)$ is given in equation (5):

$$\dot{x}(t+h) = x(t) + h\dot{x}(t) + \sum_{n=2}^{\infty} \frac{h^n}{n!} x^{(n)}(t) \tag{5}$$

Neglecting the higher terms of equation (5) becomes as follows:

$$\dot{x}(t+h) = x(t) + h\dot{x}(t) \tag{6}$$

If the quantum size, *D*, is fixed where D is given by equation (7):

$$D = |x(t+h) - x(t)| \tag{7}$$

then, the change in time, h, required for a change of size D to occur in x(t) is approximately the one in equation (8):

$$h = \begin{cases} \dfrac{D}{|\dot{x}(t)|} & \text{if } \quad \dot{x}(t) \neq 0 \\ \infty & \text{otherwise} \end{cases} \tag{8}$$

D can be considered as the resolution of the phase space grid, while h approximates the time at which the solution jumps from one phase space grid point to the next [12].Therefore it follows that:

$$\begin{aligned} h\dot{x}(t) &= hf(x(t)) \\ &= \begin{cases} \dfrac{D}{|f(x(t))|} f(x(t)) & \text{if } \quad \dot{x}(t) \neq 0 \\ \infty & \text{otherwise} \end{cases} \\ &= \begin{cases} D\,\mathrm{sgn}(f(x)) & \text{if } \quad \dot{x}(t) \neq 0 \\ \infty & \text{otherwise} \end{cases} \end{aligned} \tag{9}$$

Therefore, the solution of the first order differential equation is given by equation (10):

$$x[q+1] = x[q] + D\,\mathrm{sgn}\big(f(x(t))\big) \tag{10}$$

where q in the above equation denotes the state index, and [q+1] is the next state index.

The time t this event occurs is shown in equation (11):

$$t[q+1] = t[q] + h \tag{11}$$

2.4 NCDEVS Solver

The NCDEVS solver implements a discrete event simulation of components that work under the natural backplane using the resistive companion method (RCM). The flow chart in Figure 2 helps us in order to explain the solver.

The NCDEVS solver begins by stamping the components into the system's equations and initializing the DEVS components. After the DEVS state variables are being calculated, we evaluate the time that is needed for a change in the quantum. Since there was a synchronization issue, the minimum time for all the calculated times needs to be found. Then we roll back all the times for each component to the minimum time as well as the state variables using linear interpolation. When the calculated minimum time is larger than the final time, then the simulation terminates, otherwise we proceed to initialize the DEVS components once again.

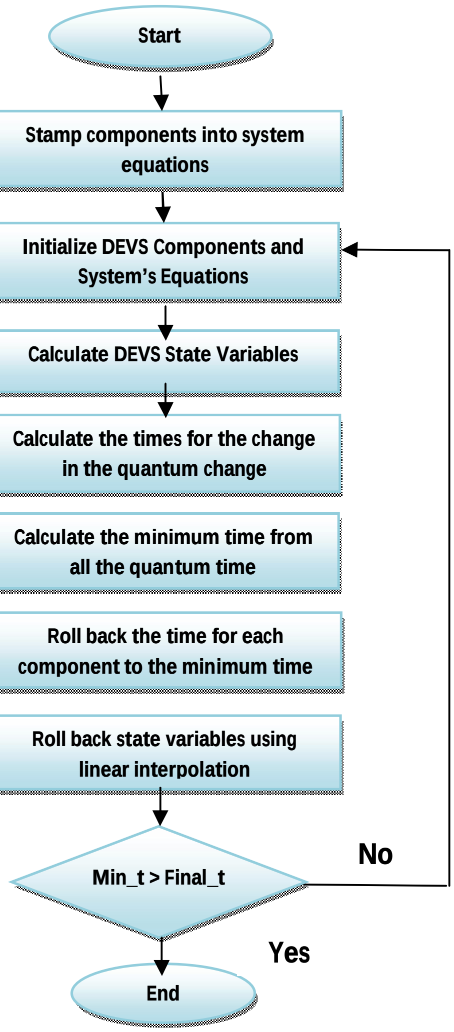


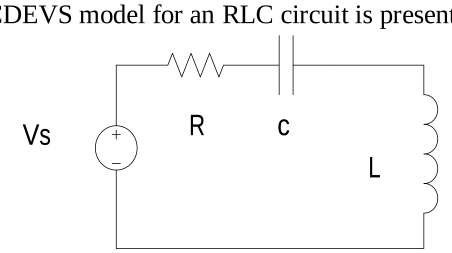
Figure 2. Flow diagram of NCDEVS solver

3. Model Derivation

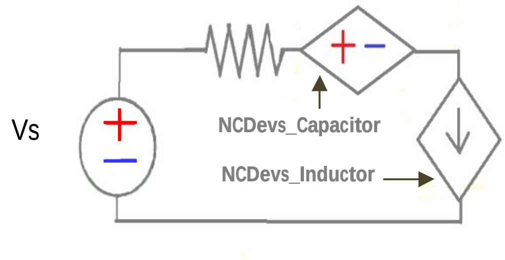
In this part of the paper, an RLC circuit was chosen to be presented that contains both types of the dynamic components. Furthermore, an NCDEVS formalization of a Permanent Magnet DC Motor (PMDC) is demonstrated.

3.1 Formalization of an RLC Circuit

The RLC analysis is presented in detailed below. In figure 3(a) there is a figure of an RLC circuit while in Figure 3(b), the NCDEVS model for an RLC circuit is presented.



(a)



(b)

Figure 3. RLC and Equivalent NCDEVS RLC

The NCDEVS inductor model is the dependent current source and the NCDEVS capacitor model is the dependent voltage source. The following part is going to explain the mathematical analysis for those representations.

The behavior of an inductor is described by equation (12):

$$f_L(t) = \frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \tag{12}$$

Using the DEVS formulation to discretize the first order differential equation, we get equation (13):

$$i_L[q+1] = i_L[q] + D\operatorname{sgn}\left(\frac{v_L(q)}{L}\right) \tag{13}$$

where the time at which the quantum change event occurs is specified by equation (14) and (15):

$$h_L = \frac{D}{|f_L(t)|} \tag{14}$$

$$t_L[q+1] = t_L[q] + h_L \tag{15}$$

Representing the inductor by its current injection, equation (13) can be stamped into the system equation as a dependent current source shown in Figure 4.

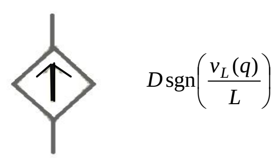


Figure 4. NCDEVS representation of an inductor

Following a similar procedure as the inductor, we were able to evaluate the model for the capacitor. We start from the governing differential equation for a capacitor which is given by equation (16):

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} \tag{16}$$

Using the DEV formulation to discretize the first order differential equation, we get equation (17):

$$v_C[q+1]=v_C[q]+D\mathrm{sgn}\left(\frac{i_C(q)}{C}\right) \tag{17}$$

The time for the quantum change in the capacitor can be calculated using equations (18) and (19):

$$h_C=\frac{D}{|f_C(t)|} \tag{18}$$

$$t_C[q+1]=t_C[q]+h_C \tag{19}$$

Equation (17) can be considered as representing a dependent voltage source shown in Figure 6, therefore a capacitor can be stamped into the system equation similarly to a dependent voltage source and is shown in Figure 5.

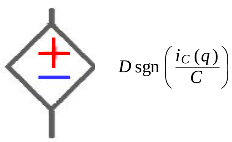


Figure 5. NCDEVS representation of a capacitor

We demonstrate the NCDEVS method, by presenting the MNA equations for the RLC circuit, which when using the NCDEVS models, it looks like Figure 3(b). The equations when solving for all the nodes are shown in equation (20).

Table 2 in the appendix summarizes the three cases of the RL, RC and RLC where it shows matrices B, G and V according to the Resistive Companion Equation.

$$\begin{aligned}
0&=\frac{Vs-V1}{R}-is\\
0&=\frac{V1-Vs}{R}-\left(i_L[q]-D*\mathrm{sgn}\left(\frac{V_L}{L}\right)\right)\\
0&=-ic+i_l[q]+D*\mathrm{sgn}\left(\frac{Vl}{L}\right)\\
0&=Vs-Vs[q+1]\\
0&=V_2-V_c(q+1)-D*\mathrm{sgn}\left(\frac{V_L}{L}\right)
\end{aligned} \tag{20}$$

3.2 Formalization of a PMDC Motor

For the formalization of a Permanent Magnet DC Motor (PMDC), we once again begin with the differential equations describing the behavior of a PMDC which is given from equation (21):

$$\begin{aligned}
f_a(t)&=\frac{di_a(t)}{dt}=\frac{k\omega(t)-R_a i_a(t)-v_a(t)}{L_a}\\
f_\omega(t)&=\frac{d\omega(t)}{dt}=\frac{T_L(t)+ki_a(t)-B\omega(t)}{J}
\end{aligned} \tag{21}$$

Using the DEV formulation to discretize the first order differential equation (22) follows:

$$\begin{aligned}
i_a[q+1]&=i_a(q)+D\mathrm{sgn}\left(\frac{k\omega(q)-R_a i_a(q)-v_a(q)}{L_a}\right)\\
\omega[q+1]&=\omega(q)+D\mathrm{sgn}\left(\frac{T_L(q)+ki_a(q)-B\omega(q)}{J}\right)
\end{aligned} \tag{22}$$

Equation (22) can be considered as an inter-connected dependent current source with a dependent voltage source. The time for the quantum change in the current and speed can be calculated using equations (23) and (24).

$$h_{ia}=\frac{D}{|f_{ia}(t)|} \tag{23}$$

$$h_\omega=\frac{D}{|f_\omega(t)|}$$

$$\begin{aligned}
t_{ia}[q+1]&=t_{ia}[q]+h_C\\
t_\omega[q+1]0&=t_\omega[q]+h_\omega
\end{aligned} \tag{24}$$

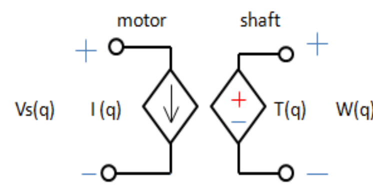


Figure 6. NCDEVS of a PMDC machine

4. SIMULATION RESULTS

We will demonstrate the simulation results using the NCDEVS method for the RLC circuit and a Permanent Magnet DC motor. Furthermore, an error analysis and a discussion of the computational intensity are provided

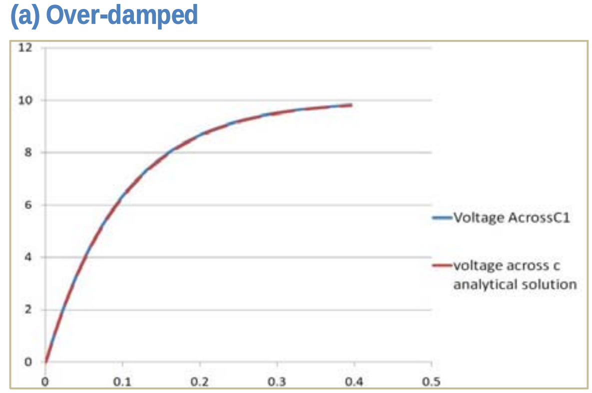
.

4.1 RLC

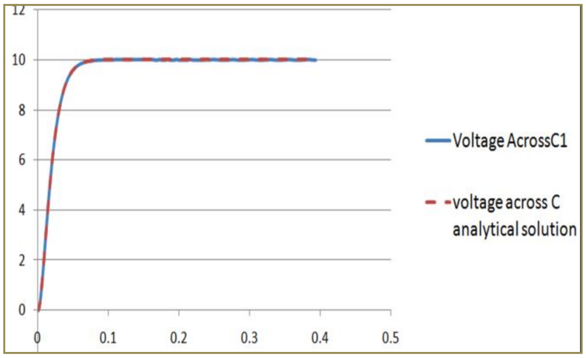
For the RLC circuit shown in Figure 3, the choice of R can determine whether the circuit is over-damped, critical-damped or under-damped. The values of L=1mH, C=0.1F, Rover-damped= 10 Ohm, Rcritical-damped= 0.2Ohm, R under-damped= 0.04 Ohm. For a fixed D = 0.001, it can be seen that NCDEV gives very accurate simulation results.

Even though our results look identical compared to the analytical solution, that is not exactly the case and therefore we performed an error analysis in chapter 4.3.

(a) Over-damped



(b) Critically-damped



(c) Under-damped

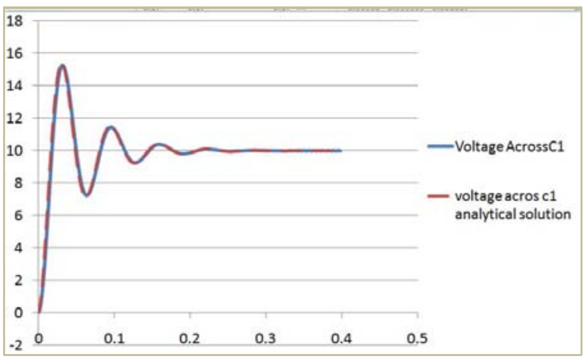
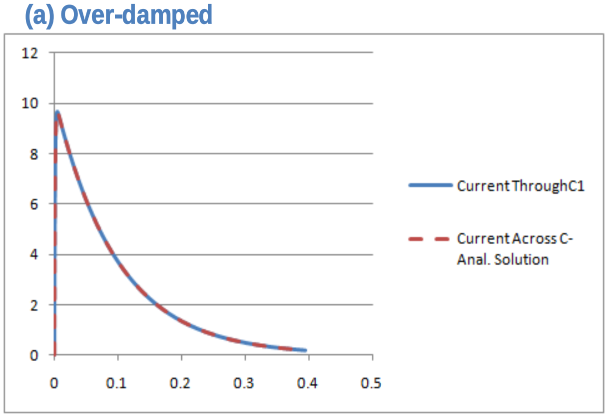
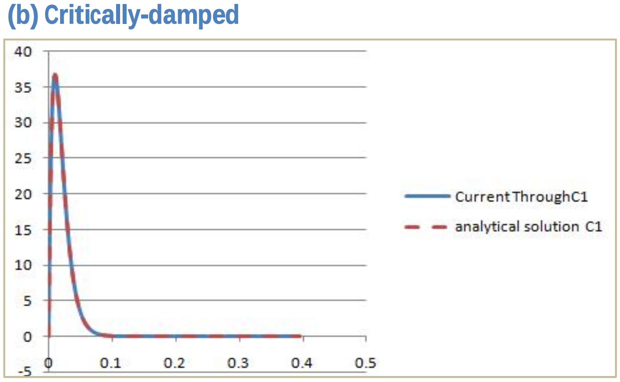


Figure 7. Comparison between the voltage across the RLC using NCDEVS and the analytical solution

(a) Over-damped



(b) Critically-damped



(c) Under-damped

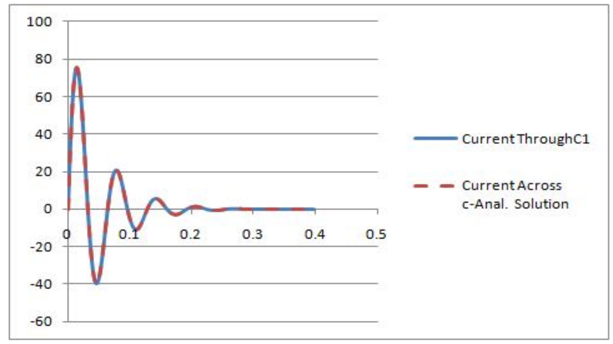


Figure 8. Comparison between the current through the RLC using NCDEVS and the analytical solution

4.2 Motor

As a second example, consider the system in Figure 9 where a half wave rectifier with an inductor and capacitor filter is loaded by a Permanent Magnet DC motor. The diode in this case is formulated using the NCDEVS scheme. Figure 10 shows the input voltage as well as the voltage across the Permanent Magnet DC.

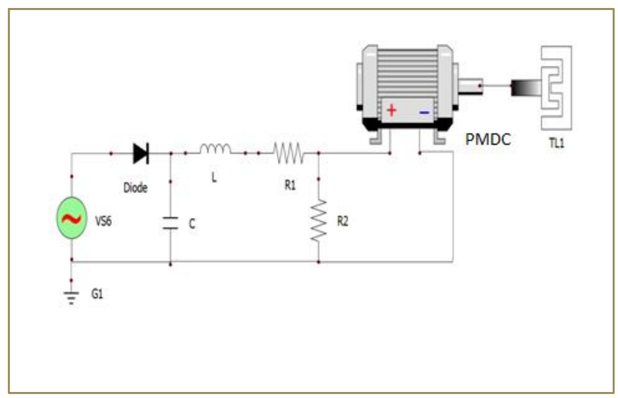


Figure 9. Half-wave rectifier circuit with an L C filter driving a PMDC motor

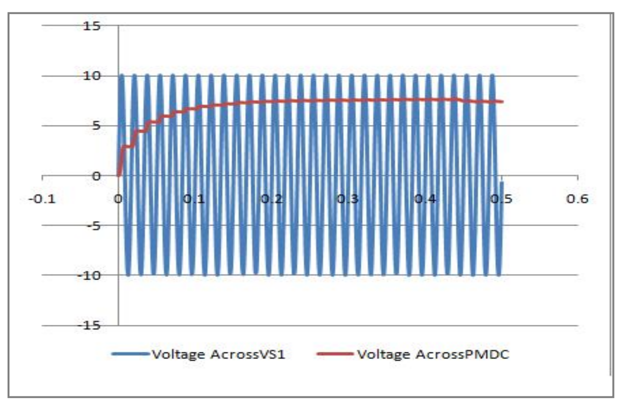


Figure 10. Simulation Results of the NCDEVS PMDC simulation

4.3 Error and Computational Intensity

In order to present the advantages of NCDES, we are going to examine different issues such as the computational intensity, the time step variation during the simulation and the error dependency on the quantization level D.

Firstly in Figure 11, we are presenting the plot of the variation of the time-step during the simulation of the PMDC. We are showing the magnified results and as it can be seen, the time step in the beginning of the simulation is quite small due to the multiple changes that arise in the state of the PMDC motor. As soon as the voltage approaches close to the steady state, the time step becomes much larger until we reach the time that we don't observe any more changes in the state, and therefore the simulation will terminate.

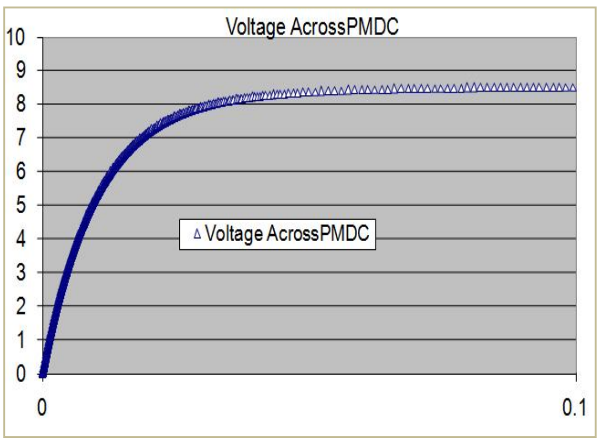


Figure 11. Simulation Results showing the time step variation

An additional issue that needs to be discussed is the error of NCDEVS compared to the analytical solution. We chose to demonstrate the case of the critically damped RLC and the comparison is shown in Figure 12. As it was expected, the error increases linearly with D as it shown in Figure 13. The smallest our choice for the quantization level D the closest our results to the analytical solution.

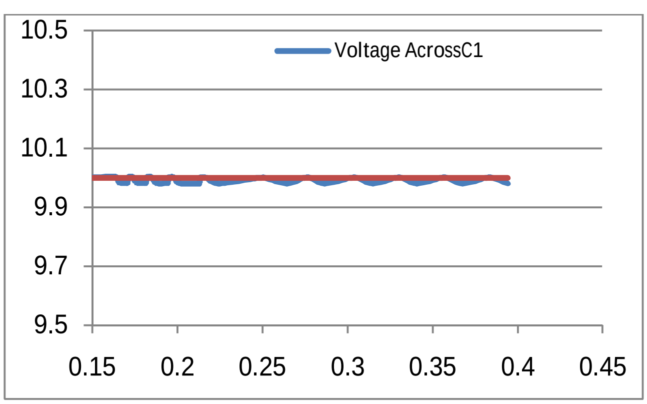


Figure 12. Simulation Results showing the error in critically damped RLC

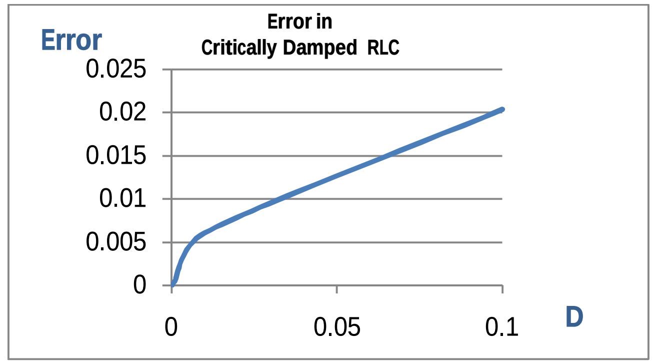


Figure 13. Simulation Results showing the error in critically damped RLC

Finally a significant benefit that NCDEVS can offer over the Resistive Companion Method (RCM) when using the trapezoidal as the integration method is the number of points of calculation. In order to have a precise comparison, we run first the RLC cases by using the RCM. Then, we chose the smallest value as our quantization D and that gave us the same time step in the beginning of the simulation. For all the cases, even though in the beginning of the simulation, the RCM showed less number of calculating points, after a certain point of time (which is different for each case), NCDEVS is the one that shows fewer number of points until the end of the simulation.

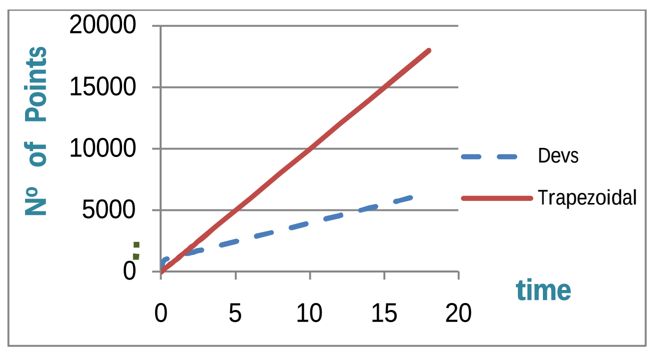


Figure 14. Simulation Results showing the number of calculation points in Overdamped RLC

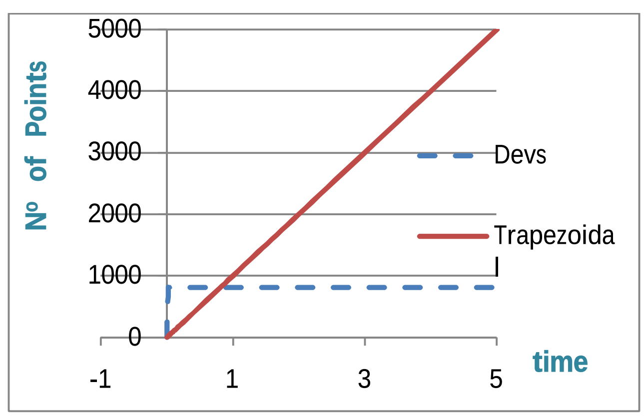


Figure 15. Simulation Results showing the number of calculation points in critically damped RLC

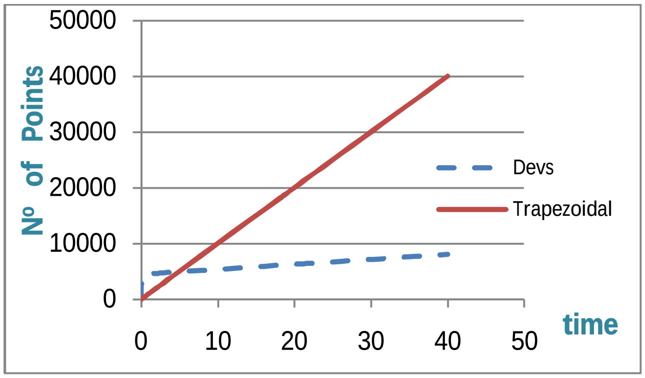


Figure 16. Simulation Results showing the number of calculation points in Underdamped RLC

5. CONCLUSION

In this paper we present a naturally coupled discrete event simulation method (NCDEVS), which is similar to the resistive companion method (RCM). By pairing natural coupling and DEVS, we created a DEVS solver that enforces the laws of conservation of energy while providing an asynchronous discretization of the simulation events. By combining the differential equation that describes each dynamic component, and by discretizing by state, we were able to create the NCDEVS equivalent models for those dynamic components. We present the mathematical equations and the equivalent circuits for the NCDEVS for basic dynamic components. By comparing the NCDEVS method with the known analytical solutions for an RLC and a PMDC motor, it is shown that our results have a small error deviation compared to the actual analytical solutions. This error depends linearly on the choice of quantization level D. In addition, the form of NCDEVS is similar to the state space format, while the simulation results have also shown that NCDEVS is specifically advantageous for cases of systems that reach steady state at a short period of time since the beginning of the simulation. NCDEVS is a multirate solver that produces direct equations. For future work, the authors are working on a solver that supports the computation of any form of current injections. In addition, the solver will able to solve an independent set of equations and not simultaneous ones. The authors recognize the singularity issues that arise when we have two controlled voltage sources connected in parallel (i.e NCDEVS capacitors), or two controlled current sources connected in series (i.e. NCDEVS inductors). The authors plan to update this new method with a valid solution in order to solve those concerns.

6. ACKNOWLEDGMENT

This work was supported by the office of Naval Office of Research under grant # N00014-08-1-0080.

Appendix

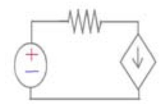
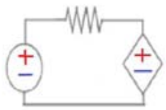
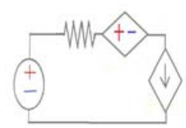
	NCDEVS MODELS	B	G	V
R L		$\begin{vmatrix} 0 \\ IL + D \times sign(\frac{VL}{L}) \\ VS \end{vmatrix}$	$\begin{vmatrix} \frac{1}{R} & -\frac{1}{R} & 1 \\ -\frac{1}{R} & \frac{1}{R} & 0 \\ 1 & 0 & 0 \end{vmatrix}$	$\begin{vmatrix} Vc^+ \\ Vc^- \\ Ic \end{vmatrix}$
R C		$\begin{vmatrix} \frac{\mathrm{Vs}}{\mathrm{R}} \\ -\frac{Vs}{R} \\ D * \mathrm{sgn}(\frac{ic}{C}) \end{vmatrix}$	$\begin{vmatrix} \frac{1}{R} & -\frac{1}{R} & -1 \\ -\frac{1}{R} & \frac{1}{R} & 1 \\ 1 & -1 & 0 \end{vmatrix}$	$\begin{vmatrix} V1 \\ V2 \\ I3 \end{vmatrix}$
R C L		$\begin{vmatrix} 0 \\ il(q) + D * \mathrm{sgn}(\frac{V_L}{L}) \\ i(q) + D * \mathrm{sgn}(\frac{V_L}{L}) \\ Vs(q+1) \\ Vc(q) + D * sg(\frac{ic}{c}) \end{vmatrix}$	$\begin{vmatrix} 1/R & -1/R & 0 & -1 \\ -1/R & 1/R & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$	$\begin{vmatrix} Vs \\ V1 \\ V2 \\ Is \\ Ic \end{vmatrix}$

Table 2. NC DEVS models for RL, RC and RLC

References

[1] Bernard Zeigler, Theory of Modeling and Simulation, John Wiley & Sons, New York, 1976.

[2] B.P. Zeigler and J.S. Lee, “Theory of Quantized Systems: Formal Basis for DEVS/HLA Distributed Simulation Environment,” SPIE Proceedings, 3369 (1998) 49-58.

[3] Kofman E. and Junco, S., “Quantized State Systems. A DEVS approach for continuous system simulation”. Technical Report LSD0004, LSD, FCEIA, UNR. To appear in Transactions of SCS, 2000. Available at www.eie.fceia.unr.edu.ar/~lsd.

[4] Cellier, F.E., E. Kofman, G. Migoni, and M. Bortolotto (2008), Quantized State System Simulation, Proc. GCMS'08, Grand Challenges in Modeling and Simulation part of SCSC'08, Summer Computer Simulation Conference, Edinburgh, Scotland, pp.504-510.

[5] Cellier, F.E. and E. Kofman (2006), Continuous System Simulation, Springer-Verlag, New York.

[6] Kofman, E., Lapadula, M. and Pagliero, E., " PowerDEVS: A DEVS–Based Environment for Hybrid System Modeling and Simulation", Technical Report LSD0306, LSD, Universidad Nacional de Rosario, 2003.

[7] Beltrame, T. and F.E. Cellier, “ Quantized State System Simulation in Dymola/Modelica Using the DEVS Formalism” , Proc. 5th International Modelica Conference, Vienna, Austria, Vol.1, 2006, pp.73-82.

[8] Chung-Wen Ho, Albert E. Ruehli, Pierce A. Brennan, “The Modified Nodal Approach to Network Analysis”, IEEE Transcactions on Circuits.

[9] T.L. Pillage, R. A. Rosher., C. Visweswariah, “Electronic Circuit and Simulation Methods”, McGraw-Hill, 1998.

[10] G. Cokkinides, B. Beker, “VTB Model Developer’s Guide”, Virtual Test Bed Project, Department of Electrical Engineering, University of South Carolina, 2001.

[11] Teems E. Lovett,”Design and Implementation of a Multi-discipline Simulation Environment”, Columbia, University of South Carolina, 2002.

[12] James Nutaro, “Handbook of Dynamic Systems Modeling”, Citeseer, 2005.

[13] Charles A. Thompson, " A Study of Numerical Integration Techniques for use in the companion circuit method of transient circuit analysis”, TR-EE 92-17, School of Electrical Engineering, Purdue University, 1992.