Elimination of Numerical Oscillations in Power System Dynamic Simulation

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Abstract—**This paper addresses numerical oscillations encountered in power system dynamic simulation resulting from trapezoidal numerical integration rule. Two methods are presented to eliminate the numerical oscillations: trapezoidal with numerical stabilizer method and Gear's second order method. A detailed comparison is given regarding the accuracy of the trapezoidal rule, trapezoidal with numerical stabilizer method, and Gear's method. The validity of the new methods is demonstrated in the simulation of a power electronic circuit within Virtual Test Bed. The new methods are of great significance in performing a meaningful simulation for power electronics circuits.**

Keywords-power system simulation; numerical oscillation; power electronics simulation; numerical integration; dynamic simulation

I. INTRODUCTION

Numerical integration of differential equations or state equations is essential for performing dynamic system simulation. There are a variety of numerical integration methods: backward Euler's, trapezoidal, Simpson's, Runge-kutta's, Gear's methods, etc. Among these methods, trapezoidal integration is the most popular one in network transient analysis due to its merits of low distortion and absolute-stability (A-stable). For example, trapezoidal integration rule is used in EMTP [2], Spice [6], and Virtual Test Bed [7], [8]. Ordinarily, dynamic equation of each circuit element is integrated using trapezoidal integration and thus the element is represented by a parallel combination of an equivalent resistance and an equivalent current source. With this modeling technique, network nodal analysis can then be performed to obtain an overall circuit simulation. This simulation methodology is usually known as Resistive Companion Form method [9], [10]. However, numerical oscillations are often encountered, especially in the simulation of power electronics circuits. Specifically, the numerical values of certain variables oscillate around the true values. In other words, only the averages of the simulated results are correct. The magnitude and frequency of these numerical oscillations are directly related to the parameters of the energy storage elements and the simulation time step. Sometimes, this problem is so severe that the simulation results are erroneous. This paper addresses this problem and its mitigation methods.

The numerical oscillations associated with trapezoidal integration result from two different reasons. One type of numerical oscillations is caused by the overly large simulation time step as compared to the smallest time constant in the network. This problem occurs especially when simulating stiff systems such as a power system with electric machines and power electronic devices. Although trapezoidal integration is absolute stable independently of time step, one must use very small time step or variable time step to avoid this numerical artifact. Another type of numerical oscillations is caused by the step changes in certain state variables. In other words, in such situations trapezoidal rule is used as a pure *differentiator*. For example, when solving voltage across an inductor after current interruption, such numerical oscillations may occur. This kind of numerical oscillations are often observed in power electronics circuits when inductive elements are present. Over the years, several approaches have been proposed to suppress this kind of numerical oscillations. One approach is to use trapezoidal rule with damping [1]. Another approach is to apply critical damping adjustment (CDA) scheme as proposed in [2], [3]. There is also a method based on wave digital filters [5].

This paper focuses on the second type of numerical oscillations associated with trapezoidal integration. A mathematical analysis for a better understanding of the mechanism of such numerical oscillations is given from two perspectives. Then, two techniques of network element modeling are presented to solve the problem of this kind of numerical oscillations. Specifically, trapezoidal with numerical stabilizer method and Gear's second order method are used. A detailed comparison is given regarding the accuracy of the trapezoidal rule, trapezoidal with numerical stabilizer method, and Gear's method. The validity of the new methods is illustrated in the simulation of a sample power electronics circuit within the Virtual Test Bed (VTB) simulation environment.

II. THE CAUSE OF NUMERICAL OSCILLATIONS

The dynamic equation of any element in power system network can be written in the following general form:

$$\frac{dx(t)}{dt} = y(t) = f(x,t) \qquad (1)$$

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where t, the time, is the independent variable; x, the state, is the dependent variable; and $f(\bullet)$ denotes a linear or nonlinear function. Let Δt be the time step for a numerical integration method and t_n $(= n\Delta t)$ be the time at n^{th} time step. In addition, let $y_n = f_n = f(x_n, t_n)$. The formula resulting from the discretization of (1) using trapezoidal rule is as follows:

$$x_n = x_{n-1} + \tfrac{\Delta t}{2}\left(y_n + y_{n-1}\right) \tag{2}$$

Often, trapezoidal integration rule is applied as *differentiator* to solve y_n given y_{n-1}, x_{n-1}, and x_n. The corresponding formula is as follows:

$$y_n = -y_{n-1} + \tfrac{2}{\Delta t}\left(x_n - x_{n-1}\right) \tag{3}$$

Suppose now that there is a step change in $x(t)$ at time instant t_k , let $x_n = x_s$ for $n \geq k$. Then it is easy to verify that the numerical solution of y_n will oscillate between $\tfrac{2}{\Delta t}\left(x_s - x_{k-1}\right) - y_{k-1}$ and $-\tfrac{2}{\Delta t}\left(x_s - x_{k-1}\right) + y_{k-1}$ for $n \geq k$. Further, the magnitude of the numerical oscillations is determined by the values of x and y at the time step immediately before the step change of x occurs, the new value of x, and the value of time step. As a result, a higher magnitude of numerical oscillations is expected if the time step is smaller.

From another perspective, trapezoidal rule can be viewed as a digital filter. For example, when it is used as a *differentiator*, the transfer function relating the input signal x_n to output signal y_n is given in the z-domain as: $\left(\tfrac{2}{\Delta t}\right)\dfrac{1 - z^{-1}}{1 + z^{-1}}$. Hence, from the theory of digital filter, the response of output signal y_n to a step change of input signal x_n suffers from sustained numerical oscillations since there is a pole at *-1* in the transfer function.

III. MODELING TECHNIQUES CAPABLE OF ELIMINATING NUMERICAL OSCILLATIONS

Two methods are considered in this paper to eliminate numerical oscillations: trapezoidal with numerical stabilizer and Gear's second order method.

"Stabilizer" denotes a conductance that is added in parallel with any "derivative" term in the dynamic model of a device. The idea is to treat the derivative term as a "voltage" across a fictitious winding. The constant in the derivative term is treated as the inductance of this winding, while the state variable that is differentiated in the derivative term is treated as the current flowing through the fictitious inductor.

For the general state equation (1), the "embedded" stabilizer can be illustrated in Figure 1. Note that, $x'(t)$ denotes the "current" flowing through the fictitious "inductor"; $g_c = \frac{\alpha\Delta t}{2L}$ is the conductance of the introduced stabilizer.

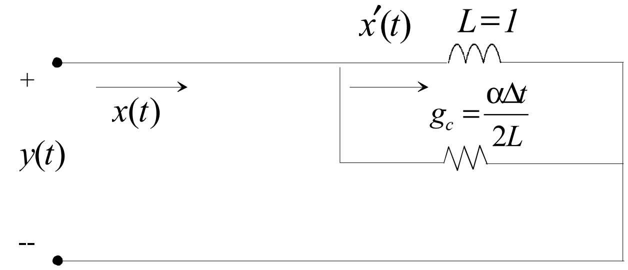

Figure 1. "Embedded" stabilizer for a general differential equation.

From Figure 1, the dynamic model equations of the device are obtained as follows:

$$y(t) = \frac{dx'(t)}{dt} \tag{4}$$

$$0 = x(t) - x'(t) - g_c y(t) \tag{5}$$

The trapezoidal rule is applied to (4) to obtain resistive companion form model, while (5) is treated as an internal equation for the internal state $x'(t)$.

Mathematically, trapezoidal with numerical stabilizer method is equivalent to the trapezoidal with damping method [1]. Parameter α corresponds to the damping coefficient, which usually takes a value between *0* and *1*. The advantage of this formulation is that the resulting modeling process is completely compatible with trapezoidal rule; i.e., the original trapezoidal rule is applied to the dynamic equations of the circuit element whose structure is slightly altered. However, contrary to intuitive perception, this alteration will not reduce the overall simulation accuracy if the stabilizer is selected appropriately. This issue is addressed in section IV of this paper.

The trapezoidal with numerical stabilizer method is capable of eliminating numerical oscillations. To prove this, its equivalent formula for discretizing (1) can be obtained after some mathematical operations:

$$x_n = x_{n-1} + \tfrac{\Delta t}{2}\left[(1+\alpha)y_n + (1-\alpha)y_{n-1}\right] \tag{6}$$

The associated transfer function when this method is used as *differentiator* is as follows:

$$H(z) = \frac{y(z)}{x(z)} = \left(\tfrac{2}{\Delta t}\right)\frac{1 - z^{-1}}{(1+\alpha) + (1-\alpha)z^{-1}} \tag{7}$$

From (7), the only pole of the discretized system of (1) with the use of this method is at $\left(\frac{\alpha-1}{\alpha+1}\right)$. Note that $-1 < \left(\frac{\alpha-1}{\alpha+1}\right) < 0$ for $0 < \alpha < 1$, indicating that the response of output signal y_n to a step change of input signal x_n oscillates but this oscillation is damped and decays in a finite number of time steps. The bigger α value, the better damping effect for eliminating the sustained numerical oscillations associated with trapezoidal rule. However, a larger local truncation error results from a bigger α value, as it is shown in section IV. It should be noted that the trapezoidal with stabilizer method corresponds to the ordinary trapezoidal integration when $\alpha = 0$, while it corresponds to the backward Euler's method when $\alpha = 1$.

On the other hand, depending on the simulation software environments, Gear's second order method may or may not be compatible with the trapezoidal rule because it is a two-step integration method while trapezoidal rule is a one-step integration method. Naturally, it requires more storage of past history data. This method was proposed in [1] as a serious contender to trapezoidal rule for power system applications. Gear's method is used in SPICE as an alternative to the default trapezoidal integration formula [6].

The formula of Gear's method for discretizing (1) is as follows:

$$x_n = \tfrac{4}{3}x_{n-1} - \tfrac{1}{3}x_{n-2} + \tfrac{2}{3}\Delta t y_n \tag{8}$$

The associated transfer function when this method is used as *differentiator* is as follows:

$$H(z) = \frac{y(z)}{x(z)} = (\tfrac{1}{\Delta t})\frac{3 - 4z^{-1} + z^{-2}}{2} \tag{9}$$

From (9), there are two poles at the origin for the discretized system of (1) after the application of Gear's method. Hence, numerical oscillations will not occur in the response of output signal y_n after a step change of input signal x_n.

Gear's second order method is A-stable for fixed time step simulation. Its accuracy is comparable to trapezoidal rule since both methods are of second order. This will be further discussed in section IV.

IV. Comparison of Accuracy

To study the accuracy of numerical computation schemes, it is important to bear in mind that any integration method results in an approximation of the true solution functions. Specifically, the obtained numerical simulation result is composed of a series of discrete values that contains only a limited number of terms of the Taylor series expansion of the continuous-time-domain solution function. Conventionally, the precision of a numerical integration method is investigated based on the truncation error analysis.

Alternatively, a numerical integration scheme can also be viewed as a digital filtering system. Regarding the dynamic system described by state equation (1), the digital filtering perspective of a numerical integration method is illustrated in Figure 2.

From Figure 2, during numerical simulation, a digital filter with the following transfer function

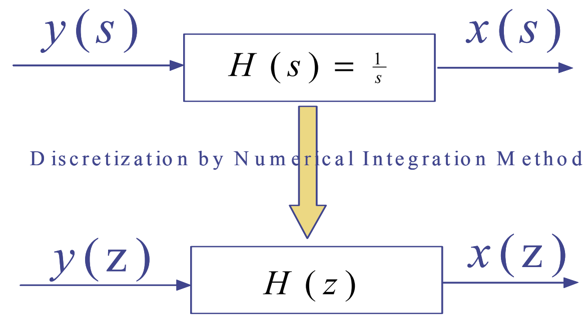

Figure 2. The digital filtering perspective of a numerical integration method.

$$H(z) = \frac{x(z)}{y(z)}, (z = e^{j\omega\Delta t}) \tag{10}$$

generates a sequence of discrete-time-domain signals x_n to approximate the true signal $x(t)$ in continuous-time-domain. The sampling frequency of the digital filter is $1/\Delta t$. Hence, the performance of each numerical integration method can be examined quantitatively by comparing the frequency response of the discrete-time system $H(z)$ to that of the original continuous-time system $H(s)$. This comparison is illustrated in Figure 3 for the trapezoidal rule, trapezoidal with numerical stabilizer method, and Gear's method. In Figure 3(a), the ratio of magnitude of $H(z)$ over magnitude of $H(s)$ is plotted against the input signal's frequency in per unit of $1/\Delta t$. If a curve is the closest to *1* for all frequencies, its corresponding numerical integration method is the most accurate in terms of the magnitudes of the simulation results. In Figure 3(b), the phase of ratio $H(z)$ over $H(s)$ is plotted against the input signal's frequency in per unit of $1/\Delta t$. If a curve is the closest to *0* for all frequencies, its corresponding numerical integration method is the most accurate in terms of the phase of the simulation results. In terms of magnitude response, the trapezoidal rule and trapezoidal with numerical stabilizer method are very accurate for frequencies up to one-fifth of the Nyquist frequency ($\tfrac{1}{2\Delta t}$). The Gear's method has higher magnitude error. In terms of phase response, the trapezoidal rule does not cause phase distortion, but the trapezoidal with numerical stabilizer method and Gear's method produce frequency-dependent phase distortion. Therefore, the trapezoidal rule seems the best in terms of numerical accuracy. But as proved in section III, it suffers from sustained numerical oscillations when used as *differentiator*. If ignoring phase distortion, the trapezoidal rule with stabilizer is more accurate than the trapezoidal rule and Gear's method (for the case of $\alpha = 0.5$). Overall, trapezoidal with numerical stabilizer method and Gear's method are best suitable for power system dynamic simulations considering their capability of eliminating numerical oscillations.

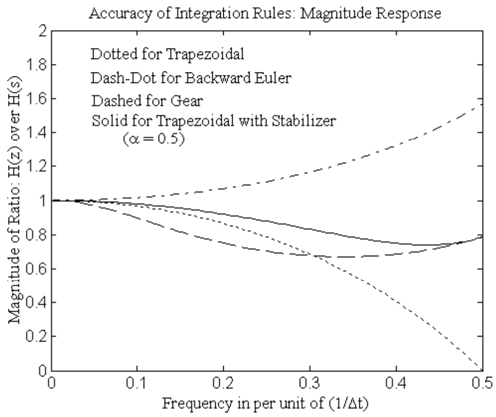

Figure 3 (a)

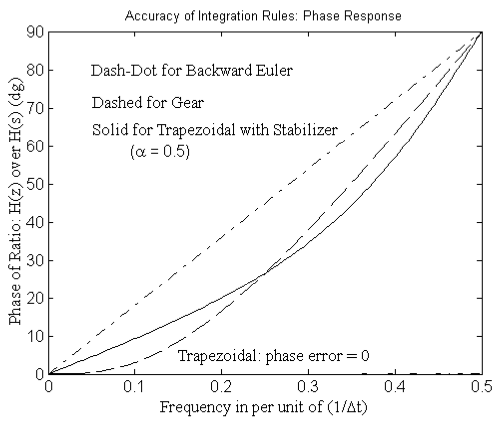

Figure 3 (b)

Figure 3. Frequency response for the integration rules. (a) Magnitude; (b) Phase.

For different values of α, ranging from *0.0* to *1.0*, the frequency responses of the trapezoidal with numerical stabilizer method are plotted in Fig. 4 for comparison. As the value of α increases as indicated by the direction of the arrows in the figure, the magnitude error decreases for $0 < \alpha < 0.5$; however, the phase error increases. In the mean time, the damping effect increases since the zero of the corresponding transfer function moves from –1 (no damping) toward zero (critical damping).

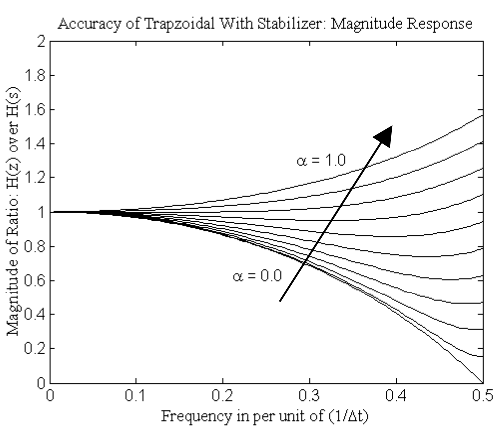

Figure 4 (a)

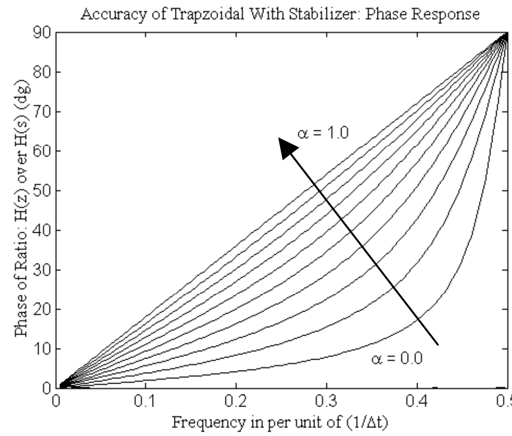

Figure 4 (b)

Figure 4. Frequency response of the trapezoidal rule with damping for different values of α. (a) Magnitude; (b) Phase.

V. IMPLEMENTATION AND VALIDATION

The new modeling techniques are implemented for several devices in Virtual Test Bed developed at the University of South Carolina, which is an interactive simulation environment that allows analysis of mixed discipline systems and provides advanced visualization capabilities. The Virtual Test Bed architecture is described in detail in [7], [8]. The validity of the trapezoidal with numerical stabilizer method and Gear's method to eliminate numerical oscillations is demonstrated in the simulation of a simple example test system within Virtual Test Bed. One advantage of VTB is that it accepts models developed with different numerical integration techniques, thus allowing compatibility among different integration methods, such as Gear's, Trapezoidal with stabilizer, ordinary Trapezoidal or other integration techniques.

The test case is illustrated in Figure 5. In this test case, a sinusoidal voltage source drives an inductive load through a diode. This scenario is often encountered in the simulation of power electronics system. The rms voltage of the voltage source is 100 volts. The total resistance is 1.1 ohms. The inductance is 10 milli-henries. The simulation time step is 10 microseconds. When the resistive companion form model of the inductor is obtained using the trapezoidal rule, the numerical oscillations in the computed voltage across the inductor are obvious, which are illustrated in Figure 6 and Figure 7. On the other hand, when the simulation model of the inductor is developed using Trapezoidal with numerical stabilizer method, there are no numerical oscillations, as it is shown in Figure 8. Same is true when the simulation model of the inductor is developed using Gear's method, as it is shown in Figure 9.

VI. CONCLUSIONS

The trapezoidal rule suffers from numerical oscillations when used for differentiation in situations such as solving voltage across an inductor after current interruption. The proposed trapezoidal with numerical stabilizer method and Gear's method are able to eliminate possible numerical oscillations. Hence, they are of great significance in performing a meaningful simulation for power electronics circuits in which switchings of semiconductor devices cause current interruptions. Both methods are very accurate for sufficiently small simulation time step. The trapezoidal with numerical stabilizer method is the best since it is fully compatible with the trapezoidal method, which has already been widely used in several power system transient analysis programs such as EMTP, SABER, SPICE, and VTB.

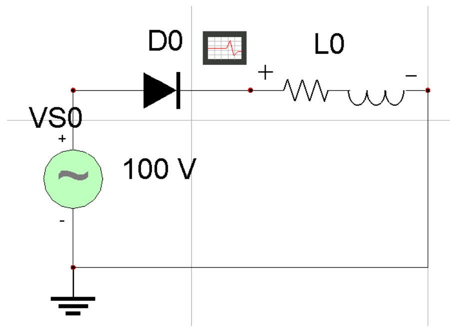

Figure 5. Test Case: The modeling of an inductor.

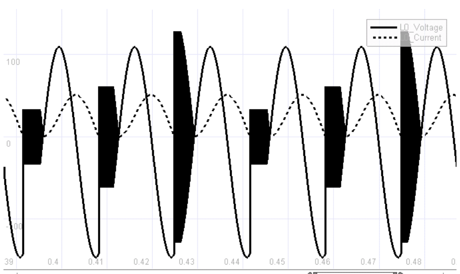

Figure 6. Numerical oscillations after current interruption.

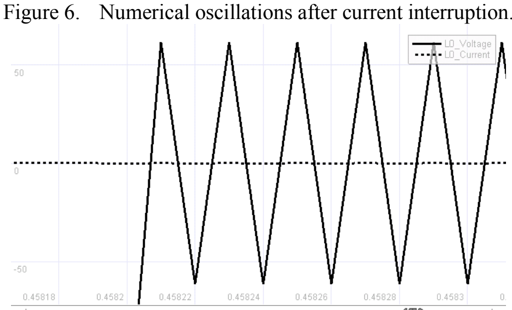

Figure 7. Zoomed view of numerical oscillations after current interruption.

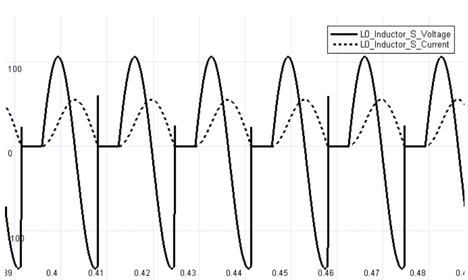

Figure 8. Elimination of numerical oscillations after current interruption using trapezoidal with numerical stabilizer method

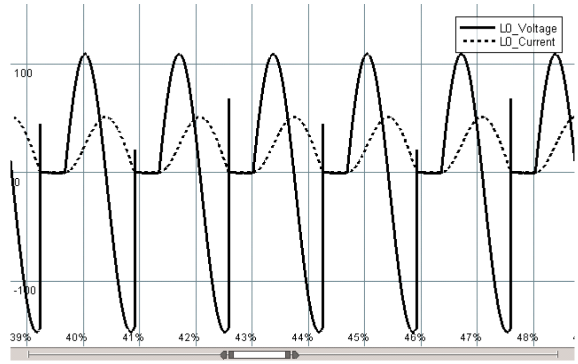

Figure 9. Elimination of numerical oscillations after current interruption using Gear's method frequent

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