A Method for Coupling Phasor and Time Domain Networks

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Abstract

A method for coupling dynamic phasor models to time domain resistive companion networks is proposed for power system simulation. The method uses controlled sources, modified nodal analysis, and a discrete Fourier transform (DFT) phasor estimation. A coupling device using this technique is implemented in Virtual Test Bed (VTB). The accuracy and stability of the method is verified with two test systems, including a system consisting of a phasor electrical network attached to a three phase induction motor. The hybrid simulations produce exact steady-state results, nearly exact results for slow transients (on the order of 10 cycles), and good approximations for faster transients (on the order of 1 cycle). Unbalanced fault conditions are also simulated with satisfactory results.

1. INTRODUCTION

Electrical transmission and distribution networks consisting primarily of simple, passive devices (lines, transformers, capacitors etc.) are best analyzed using phasor techniques, while the electromechanical transients associated with ac generators, motors, and their controls require time domain models. Because the time constants of mechanically-dominated transients are typically larger than the fundamental electrical period, a quasisteady-state approximation is often used that allows co-simulation of a phasor electrical system with time domain induction motor models. This type of coupling makes feasible the simulation of electromechanical transients associated with machines connected to very large electrical networks. However, there are several significant limitations to the quasisteady-state approximation. When the speeds of the transients of interest begin to approach the fundamental electrical frequency, these coupled system models become inaccurate. Also, the machines are typically represented using reduced-order models that do not enforce energy-conservation at the interface between the phasor and time domain systems.

Methods have been proposed as alternatives to the quasisteady-state approach. Dynamic Phasor techniques are both more computational efficient than a full time domain approach, and more accurate than the quasisteady-state approach. Models using Dynamic phasor techniques have been shown to perform extremely well in simulating electromechanical and electromagnetic machine transients [1], and these methods have been successfully extended for use with polyphase systems and machines [2]. The disadvantage of dynamic phasor methods is the mathematical complexity of the model derivation process, and the lack of a systematic method of coupling devices together. Methods using harmonic domain concepts successfully simulate hybrid systems using iterative algorithms that minimize harmonic current mismatches at a coupling node between linear and non-linear networks [3][4]. However, these methods only solve for the periodic transients of the coupled system in the harmonic domain, and cannot provide any information about non-periodic or transient events.

This paper proposes a new approach to bridging the gap between phasor and time domain analysis. The proposed solution is a generic coupling device that couples nodes between generic phasor and time domain networks. These coupled networks may contain models of arbitrary complexity and non-linearity, as long as these models can be described in the resistive companion form (RCF). Many well studied and time tested models exist for electrical machines in the time domain. With a suitable phasor and time domain coupling method, these time domain device models can be used along with phasor electrical network models without the need to convert them into reduced-order, approximated models. Also, because the RCF is able to model behavior of arbitrary complexity, including non-linear behavior, there are few limits to the types of time domain models that can be used with this coupling method.

In Section 2, a brief introduction to the RCF method is given, and the method is generalized for use with phasor networks. Section 3 describes the proposed coupling technique. The performance of the technique is demonstrated in Section 4, first with a simple linear circuit,
and then with a full-order phase domain model of an induction motor as it operates through the startup transient, and then as it operates during an unbalanced fault condition.

2. RESISTIVE COMPANION PHASOR NETWORK

2.1. Resistive Companion Form

The RCF modeling technique allows device models to be developed that can easily be interconnected while enforcing energy conservation laws at device connection terminals. The technique is described in [5], and is extended for non-linear and time-varying modeling in [6]. The device equations must be expressed as set of Norton equivalent branches between the external device terminals and any internal nodes. The device equations are described in the form

\[
\begin{bmatrix}
    i(t) \\
    0
\end{bmatrix}
= \begin{bmatrix}
    g_{11} & g_{12} \\
    g_{21} & g_{22}
\end{bmatrix}
\times
\begin{bmatrix}
    v_1(t) \\
    v_2(t)
\end{bmatrix}
- \begin{bmatrix}
    b_1(t-h) \\
    b_2(t-h)
\end{bmatrix}
\]  

(1)

where \(i(t)\) is the set of current injections into the device terminals, \(v_1(t)\) is the vector of internal node across values (Note that the current injections for internal nodes sum to 0). The \(G\) matrix contains the partial derivatives of the \(i(t)\) vector with respect to the \(v(t)\) vector (these are conductance values for linear devices). The \(B\) vector contains the contributions from the past history of state variables, higher order expansions of nonlinear functions, and/or contributions from external controls. The form shown in (1) is a linearized version of the more general form from [5], but the form in (1) is suitable for this discussion.

A system comprised of a set of interconnected RCF models may be solved by stamping the \(G\) and \(B\) matrices of all connected devices into a single set of network \(G\) and \(B\) matrices, based on the connections between device terminals. The system is then solved for the across values at each node by solving the full network form of (1) with any valid method. An iterative solution may be used if the device equations are non-linear.

2.2. Generalization for a Phasor Network

We will now generalize the RCF method for a system with complex-valued across and through variables. This allows the simulation of single frequency phasor devices. In general, the devices may be nonlinear, and time dependant. The device equations for a phasor RCF device are in the form

\[
\begin{bmatrix}
    I(\omega, t) \\
    0
\end{bmatrix}
= \begin{bmatrix}
    G_{11}(\omega) & G_{12}(\omega) \\
    G_{21}(\omega) & G_{22}(\omega)
\end{bmatrix}
\times
\begin{bmatrix}
    V_1(\omega, t) \\
    V_2(\omega, t)
\end{bmatrix}
- \begin{bmatrix}
    B_1(\omega, t-h) \\
    B_2(\omega, t-h)
\end{bmatrix}
\]  

(2)

where all variables are complex-valued versions of those in (1). The device \(G\) and \(B\) values may be parameterized by the phasor system frequency to allow for frequency-dependant simulations. The phasor RCF network can be built and solved the same way as for the real-valued RCF network. The network can be simulated with either time, frequency, or both as independent variables, depending on the behavior modeled and the desired results. A typical, simple application is a sinusoidal steady-state model of an inductor. The RCF equation for this two port device is

\[
\begin{bmatrix}
    I_1(t) \\
    I_2(t)
\end{bmatrix}
= \begin{pmatrix}
    \frac{1}{r+j\omega L} & 1 \\
    -1 & 1
\end{pmatrix}
\times
\begin{bmatrix}
    V_1(t) \\
    V_2(t)
\end{bmatrix}
\]  

(3)

where \(r\) and \(L\) are the series resistance and inductance of the inductor. Non-linear and phasor-dynamic behavior can also be modeled with the phasor RCF technique, using series expansions and numerical integration.

3. PHASOR AND TIME DOMAIN COUPLING

3.1. Problem Statement

The behavior of coupling should be such that the instantaneous value of the modulated voltage at the phasor node is equal to the instantaneous value of the voltage at the time domain node. Also, the instantaneous value of the modulated current flowing out of the phasor node and the instantaneous value of the current flowing out of the time domain node, must sum to zero. In other words, at the point of coupling, KCL and KVL must apply to the set that includes the instantaneous values of the time domain signals and the modulated phasor signals.

If the coupling device is a “black box”, with no knowledge \textit{a priori} of the topology or dynamics of the coupled systems, the coupling cannot be ideal, because the amplitude and phase of a time signal cannot be determined without some amount of past history samples from the time signal. Therefore, the behavior of a real coupling device is described by

\[
i(t) + i(t) \sin(\omega_0 t + \theta_i(t)) + i_e(t) = 0
\]

\[
v(t) = V(t) \sin(\omega_0 t + \theta_v(t)) + v_e(t)
\]  

(4)

(5)

\[\text{Figure 1. Phasor to time domain coupling.}\]
where \( i_e(t) \) and \( v_e(t) \) are the error of the current and voltage signals. The problem, therefore, is to implement a coupling device as shown in fig. 1 that minimizes the magnitude and duration of the error contributions \( i_e(t) \) and \( v_e(t) \) in (4) and (5); such that the simulation of the coupled systems remain stable and the simulation accuracy is adequate for the behavior being modeled.

3.2. Proposed Coupling Technique

The proposed coupling device is shown in fig. 2. The device contains three nodes; an external phasor port, an external time-domain port, and an internal time-domain port. The external ports are for interconnection to their respective networks.

Behind the phasor port is an ideal current source having amplitude and phase controlled by the demodulated current value measured at the time domain port. Behind the time domain port is a modified nodal analysis gyrator circuit that creates an ideal voltage source. This voltage source is controlled by a modulated phasor value measured at the phasor port.

The RCF stamps describing the phasor and time domain subsystems of the device are

\[
G_{\text{time}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B_{\text{time}} = \begin{bmatrix} v(t) \sin(2nf_t + \theta_v(t)) \\ 0 \end{bmatrix}
\]

(6)

\[
G_{\text{phasor}} = 0, \quad B_{\text{phasor}} = \text{demod}(i(t)).
\]

(7)

where the subscripts \( \text{time} \) and \( \text{phasor} \) denote the time domain and phasor subsystems of the coupling device. This coupling solution has the following features:

- The phasor system Jacobian \( (G) \) matrix is not changed after the simulation is started, and therefore only has to be inverted once per simulation (if all phasor devices are time-invariant).
- By using MNA, there are no dissipative elements and the coupling device is lossless.
- The voltage modulation occurs instantaneously, and therefore does not introduce any error into the coupling loop. The only source of error in the coupling loop is the demodulation of \( i(t) \).

3.3. Current Phasor Estimation

Demodulation of the \( i(t) \) signal requires fast and accurate detection of the phase and amplitude of \( i(t) \) with respect to a single known frequency. Phase-locked loop (PLL) algorithms solve this sort of problem so we explored several such methods. The fastest and most stable PLL algorithm we tested in this application is proposed in [7]. However, even though this PLL algorithm avoids the instability pitfalls of common PLL algorithms, when used in our coupler it easily became unstable with certain test systems (particularly with extremely under-damped systems). Because of problems with stability of PLL algorithms, we next explored DFT detection methods.

In choosing a suitable DFT phasor estimator, we considered that a full STFT or FFT is not necessary because only one frequency component of the signal is needed. A fast and simple method of detecting a single frequency is the Goertzel algorithm, described in [8], which quickly computes only one specific Fourier coefficient of a time domain signal. The Goertzel algorithm is almost exclusively used in DTMF tone detection, but it has also been successfully used for phasor detection with power system applications [9].

The derivation of a computationally efficient sliding Goertzel DFT filter is described in [10]. The transfer function for this filter is

\[
H[z] = \frac{1 - e^{-j\frac{2\pi k}{N}z^{-1}}}{1 - 2\cos\left(\frac{2\pi k}{N}\right)z^{-1} + z^{-2}}
\]

(8)

Figure 2. The proposed coupling technique.

Figure 3. Sliding Goertzel filter implementation.
where \( N \) is the sample size and \( k \) is the index of the frequency coefficient of interest. The implementation of the filter used in the coupling algorithm is shown in fig. 3, where the output of the filter \( (y[n]) \) is properly scaled and converted into the phasor form. Of course, the output of the filter is not valid until all the points of the window have been sampled, however, after a step change in the amplitude and/or phase of the input signal, the output of the filter tends to ramp towards the steady-state result. With careful control of the sampling frequency and window size, the amplitude and phase detection can be fast and exact in steady-state with a computational complexity of \( O(1) \).

The window size \( N \) should be chosen based on the simulation time step and system frequency in order to minimize detection time and spectral leakage. Because the choice for the frequency index \( k \) is not limited to an integer for the Goertzel filter, the window does not need to span an entire cycle of the ac waveform. Using \( k \) equals 0.5, exact phasor estimation can be achieved in one half cycle. Also, there is no requirement (or increase in efficiency) for \( N \) values that are powers of two for the Goertzel filter. The calculated filter parameters for 60 hertz detection are shown in table I. These parameters allow a fast, exact solution in steady state with no spectral leakage.

The performance of the DFT demodulator was tested by applying a 60 hertz sine wave with amplitude of one and an initial phase of zero radians. The response to amplitude and phase step changes is shown in fig. 4. Phase and amplitude transients during the half cycle periods following the disturbances are smooth enough that, in practice, they do not cause instability in the coupling loop.

An alternative half cycle phasor estimation algorithm is presented in [11], which is very stable and handles dc offsets extremely well. However, this method is not well suited for the proposed coupling technique because the estimated values for the first half cycle oscillate too much during convergence. No suitable phasor estimation methods were found in the literature that converged within one half cycle.

4. VALIDATION OF COUPLING METHOD

The proposed coupling method was implemented as an entity in VTB [12] using the C# language. The icon representing the device is shown in fig. 5. The square shaped port represents the phasor connection, and the circle shaped port is the time-domain connection. Two test systems were created. The first is a simple impedance divider circuit, and the second is a three phase phasor electrical system attached to an induction motor load.

4.1. Test System 1: Hybrid Impedance Divider

Figure 6 shows a hybrid phasor/time domain circuit that uses the proposed coupling technique. The left side of the circuit is a phasor source in series with a phasor impedance. The right side is a time domain branch with a resistor and inductor. The system parameters are listed in table 2.

The comparisons to be made in validating the results are, first, variables from the time domain portion of a hybrid test system should be compared with variables from a full time domain simulation of the test system. For example, \( V_{out} \) from the hybrid system of fig. 6 should be compared to \( V_{out} \) from a full time domain equivalent of the system. Secondly, KCL enforcement within the device should be verified by comparing the current into and out of the device (labeled as \( I_{phasor} \) and \( I_{time} \) in fig. 6). In order to test convergence with

<table>
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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
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<tr>
<td>( f_0 )</td>
<td>Phasor System Frequency</td>
<td>60 Hz</td>
</tr>
<tr>
<td>( f_s )</td>
<td>Sampling Frequency</td>
<td>1200 Hz</td>
</tr>
<tr>
<td>( N )</td>
<td>Window Size</td>
<td>10</td>
</tr>
<tr>
<td>( k )</td>
<td>Frequency Index</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1. Sliding Goertzel filter parameters.

Figure 4. DFT demodulator performance with amplitude step from 1 to 2 at 100 ms and phase step from 0 to \( \pi/2 \) rad at 140 ms.

Figure 5. VTB coupling device interface
poor initial conditions, the initial values of the DFT phasor estimator were set to zero at the start of the simulation. Three current values are plotted together in fig. 7, the current obtained from the full time domain reference simulation, the current out of the coupling device (\(I_{\text{time}}\)) and the current into the coupling device (\(I_{\text{phasor}}\)) in the hybrid simulation. The currents converge very quickly. The error of \(V_{\text{out}}\) in the hybrid simulation is very small, and converges to zero within one cycle (fig. 8).

4.2. Test System 2: Induction Motor Load

The second test system represents a three phase induction motor load attached to the end of a cable connected to a phasor bus. Also attached to the bus is a constant impedance load and a generator connected through a cable. The system is shown in fig. 9. The motor model used is a high order phase domain induction machine model. The derivation and validation of this model is found in [13]. The system parameters are listed in table 3. A full time domain version of the hybrid system in fig. 9 was created for comparison. The controlled switches are open at the start of the simulation. At time equals 0.1 seconds, all three phases are switched in. The motor reaches rated speed in a little under one second. The motor speed response computed by the hybrid system is nearly identical to that computed by

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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</thead>
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<tr>
<td>(V_{\text{in}})</td>
<td>Phasor System Voltage Source</td>
<td>10 (\angle 0^\circ) V</td>
</tr>
<tr>
<td>(R_1)</td>
<td>Phasor Branch</td>
<td>1 + j2(\pi f_0)0.01</td>
</tr>
<tr>
<td>(R_2)</td>
<td>Time Domain Branch Resistance</td>
<td>10</td>
</tr>
<tr>
<td>(L_2)</td>
<td>Time Domain Branch Inductance</td>
<td>0.002 H</td>
</tr>
<tr>
<td>(f_0)</td>
<td>System Frequency</td>
<td>60 Hz</td>
</tr>
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</table>

Figure 6. VTB test system 1

Table 2. VTB test system 1 parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>(V_{\text{in}})</td>
<td>Phasor System Voltage</td>
<td>10 (\angle 0^\circ) V</td>
</tr>
<tr>
<td>(R_{\text{a}}, R_{\text{b}}, R_{\text{c}})</td>
<td>Per Phase Phasor Load Resistance</td>
<td>0.4 (\Omega)</td>
</tr>
<tr>
<td>(Z_L)</td>
<td>Mechanical Load</td>
<td>5 + j2(\pi f_0)0.002 (\Omega)</td>
</tr>
<tr>
<td>(T_m)</td>
<td>System Frequency</td>
<td>0.01 p.u.</td>
</tr>
<tr>
<td>(f_0)</td>
<td>System Frequency</td>
<td>60 Hz</td>
</tr>
<tr>
<td>(R_s)</td>
<td>Per Phase Motor Stator Resistance</td>
<td>0.531 (\Omega)</td>
</tr>
<tr>
<td>(L_p, L_r)</td>
<td>Per Phase Motor Stator and Rotor Inductances</td>
<td>252 mH</td>
</tr>
<tr>
<td>(L_m)</td>
<td>Per Phase Motor Magnetizing Inductance (Referred to Stator)</td>
<td>847 mH</td>
</tr>
<tr>
<td>(R_p)</td>
<td>Per Phase Motor Rotor Resistance</td>
<td>0.408 (\Omega)</td>
</tr>
<tr>
<td>(V_{\text{rated}})</td>
<td>Motor Rated Voltage</td>
<td>220 V</td>
</tr>
<tr>
<td>(P_{\text{rated}})</td>
<td>Motor Rated Power</td>
<td>3570 W</td>
</tr>
<tr>
<td>(T_{\text{rated}})</td>
<td>Motor Rated Torque</td>
<td>19.4 N (\cdot) m</td>
</tr>
</tbody>
</table>

Figure 7. Test system 1 currents

Figure 8. Test system 1 Vout voltage

Table 3. VTB test system 2 parameters
**Figure 9.** VTB test system 2

**Figure 10.** Motor shaft speed at startup

**Figure 11.** Stator current phase A at startup
the full time-domain reference model, as seen in fig. 10. The largest speed deviation between simulations occurs in the speed ripple during the first few cycles after the switches are closed (fig. 10.b), but this deviation is very small.

The stator currents from both simulations appear to be exactly the same in steady-state (fig. 11.a) and the fast startup transient is also very accurately represented (fig. 11.b). An interesting result is that, because the phasor detector introduces a delay of about one half-cycle, the decaying positive dc current offset in the stator current is converted to a negative dc offset in the phasor current in the first few cycles. The plot in fig. 11.c shows the current response as the motor reaches rated speed, and it highlights the excellent performance of the coupling method for slower transients.

An important result from the simulation of a motor at startup is the transient voltage drop of the connected electrical system. Like the current response, the voltage response contains both fast and slow transients. The steady-state and slow transient can be seen in fig. 12.a, showing about a 10% voltage drop during the time that the motor accelerates to rated speed. The fast transient response is plotted in fig. 12.b, along with the voltage amplitude envelope computed by the hybrid simulation. Again, like the current, the magnitude and shape of the fast voltage transient appears to match that of the full time domain simulation, but is slightly delayed.

Unbalanced behavior of electrical machines is particularly difficult to model with phasor analysis. It is an interesting exercise, therefore, to determine if the proposed
The speed response shows a proper estimation of the speed ripple (2nd harmonic) with no delay (fig. 14). However, the slow dc speed change is slightly delayed. This delay error goes away around 2.5 seconds (not shown in plot), as the dc speed begins to converge on its final value. The torque response in the hybrid simulation predicts the fast and slow torque transients and the 2nd harmonic ripple remarkably well (fig. 15). The current response, plotted in fig. 16, shows all current values converging within 1 cycle, even with a very abrupt change in the current angle.

5. CONCLUSION

A new method has been proposed for coupling phasor and time domain simulation models of electrical networks. The method is shown to be accurate and stable for a simple linear hybrid circuit, and also for a more complicated case of a phasor electrical system coupled with a time domain induction motor operating through startup and unbalanced fault conditions. The method is significantly different from the methods previously proposed in [3] and [4], especially in that it simulates non-periodic transients. The proposed method also allows for the reuse of a wide range of existing models that use the versatile resistive companion form.

One possible application of the coupling method is for electromechanical transient stability and voltage stability analyses of large generators and motors interconnected through vast electrical networks. Future work may include a more rigorous stability analysis and stability improvements based on that analysis. Also, this coupling method may be extended to work with a harmonic domain resistive companion network solver. This would allow harmonic coupling between periodic and time domain networks, making the coupling method suitable for use in the simulation of switched power devices, transformers in saturation, and in other applications where system responses to significant harmonics distortion is of interest.

References