A Novel Passivity-Based Stability Criterion (PBSC) for Switching Converter DC Distribution Systems

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Abstract—A novel Passivity-Based Stability Criterion (PBSC) is proposed for the stability analysis and design of DC power distribution systems. The proposed criterion is based on imposing passivity of the overall bus impedance. If passivity of the overall DC bus impedance is ensured, stability is guaranteed as well. The PBSC reduces artificial design conservativeness and sensitivity to component grouping typical of existing stability criteria, such as the Middlebrook criterion and its extensions. Moreover, the criterion is easily applicable to multi-converter systems and to systems in which the power flow direction changes, for example as a result of system reconfiguration. Moreover, the criterion can be used for the design of active damping networks for DC power distribution systems. The approach results in greatly improved stability and damping of transients on the DC bus voltage. Experimental validation is performed using a hardware test-bed that emulates a DC power distribution system.

I. INTRODUCTION

In recent times DC power distribution systems consisting of a network interconnection of feedback-controlled switching power converters are being proposed as an alternative to conventional AC distribution systems in industrial applications [1-2] and in military applications such as the power distribution system for the all-electric ship proposed by the US Navy [3, 4, 17]. Advances in power electronics technology and the impact of renewable distributed generation sources make a DC distribution system an attractive solution due to high reliability, interface flexibility, and high power density. However, one the challenges for a DC distribution system is a degradation of stability due to interactions among feedback-controlled converters. Typically, feedback-controlled converters behave as constant power loads (CPLs) at their input terminals within the bandwidth of the system control loop gain [5, 6]. CPLs exhibit negative incremental input impedance, which is origin of the undesired destabilizing effect and cause of the subsystem interaction problem [6]. Although each subsystem is independently designed to be standalone stable, a system consisting of many power-electronics-based subsystems may exhibit degraded stability due to subsystem interactions caused by CPL behavior.

Existing stability analysis criteria, like the Middlebrook criterion [7], and its various extensions, such as the Gain and Phase Margin criterion, the Opposing Argument criterion [8], and the ESAC criterion [9, 10], impose stability conditions on the load-impedance/source-impedance ratio. The Middlebrook criterion states that a sufficient condition for system stability is that the Nyquist contour of that impedance ratio lies within the unity circle. An advantage of the Middlebrook criterion is that it is design oriented. Its main disadvantage is that it leads to artificial conservativeness in the design of DC distribution systems. In particular, it can be shown that stability is still preserved, even though the Nyquist contour of the impedance ratio lies outside the unity circle, as long as some phase constraint is satisfied. For this reason, extensions to the Middlebrook criterion, such as the Gain and Phase Margin criterion, and the Opposing Argument criterion, were proposed. These last two methods, even though less conservative than the Middlebrook criterion, encounter difficulties when applied to multi-converter systems (with more than two interconnected subsystems) – especially in the case where power flow direction changes – and are sensitive to component grouping [10]. As an attempt to reduce all these problems, the ESAC criterion and its refinement, the Root Exponential Stability Criterion (RESC) [10], were more recently proposed. The ESAC criterion can be used to determine total load admittance specifications when a source...
In the present work a novel Passivity-Based Stability Criterion (PBSC) is proposed. The method imposes a passivity condition on the overall bus impedance rather than imposing conditions on an impedance ratio. This leads to several advantages:

- the PBSC can easily handle multiple interconnected converters and inverter of power flow direction.
- the PBSC reduces artificial design conservativeness and sensitivity to component grouping typical of all prior stability criteria.
- the criterion lends itself to the design of active damping impedances for DC power distribution systems. In particular, the method can be coupled with a recently proposed control strategy for switching converters called Positive Feed-Forward (PFF) control [11-13], to provide a control design method that ensures overall system stability and performance.

The stability criterion is introduced in Section II and illustrated in a simple example in Section III. In Section IV a second more complex example of a DC power distribution system is discussed. The PBSC criterion is used for the design of an active damping controller for the switching inverter connected to the DC bus. Experimental results are presented validating the DC bus stabilization obtained with the method. Conclusions are given in Section V.

II. PASSIVITY-BASED STABILITY CRITERION

In this section necessary and sufficient conditions are given for the stability of two (or more) interacting subsystems being part of a larger DC power distribution system, as depicted in Fig. 1.

Before the proposed stability criterion is given, let us recall the definition of passivity of a 1-port linear electrical network described by impedance \( Z(s) = V(s)/I(s) \). The network is passive if it can only absorb energy. In mathematical terms, an electrical network is passive if and only if \( \int_{-\infty}^{+\infty} v(t)i(t)dt \geq 0 \) for all \( T \). The total energy delivered to the network is

\[
\int_{-\infty}^{+\infty} v(t)i(t)dt = \\
= \frac{1}{2\pi} \int_{-\infty}^{+\infty} V(j\omega)I^*(j\omega)d\omega \\
= \frac{1}{2\pi} \int_{-\infty}^{+\infty} Z(j\omega)||I(j\omega)||^2d\omega \\
= \frac{1}{\pi} \int_{0}^{+\infty} Re[Z(j\omega)||I(j\omega)||^2d\omega
\]

where Equations (1)-(2) use Parseval’s Theorem reported in the Appendix. Passivity condition requires Equation (4) to be non-negative for all possible currents, as previously stated. This implies that \( Z(s) \) which is a real, rational function of \( s \) must be a positive real function, or \( Re[Z(\sigma + j\omega)] \geq 0 \) for \( \sigma > 0 \). The latter condition could be very complex because it requires the computation of \( Re[Z(s)] \) at each frequency in the right half \( s \)-plane (RHP). However, the following necessary and sufficient passivity condition provides a solution to this computation difficulty. A linear time invariant 1-port is passive [15] if and only if:

1. \( Z(s) \) has no right half plane (RHP) poles,
2. \( Z(j\omega) \) has a Nyquist contour which lies wholly in the closed RHP.

The second condition comes from the application of the principle of maximum modulus [16] to a Nyquist contour over the RHP. In order to apply this principle, the Nyquist contour of \( Z(j\omega) \) cannot enclose any poles, hence condition 1. As a result, RHP poles (and zeros as well) are prohibited. A direct consequence of condition 2 is that current and voltage cannot be more than 90° out of phase at any frequency [14]. Thus the Nyquist contour of \( Z(j\omega) \) cannot enter the left half plane (LHP). In other words, the phase of \( Z(j\omega) \) must be between \(-90° \) and \(+90° \) at all frequencies. The phase of \( Z(j\omega) \) is the difference between the phase of the voltage \( V(j\omega) \) applied to the port and the phase of the current \( I(j\omega) \) injected into the port. If the phase of \( Z(j\omega) \) is between \(-90° \) and \(+90° \) for every \( \omega \), the average power into the port is positive for all possible currents and therefore the system consumes energy (it is a passive system). If the phase is \(+90° \) or \(-90° \) for every \( \omega \), the average power is zero, and the system is lossless. If the phase is less than \(-90° \) or greater than \(+90° \) for some \( \omega \), the average power can be negative and the system can produce energy (it is an active system).

The proposed Passivity-Based Stability Criterion (PBSC) for switching converter DC distribution systems (Fig. 1) states that:

- If passivity (and therefore the phase constraint) is satisfied for \( Z_{tot}(s) = Z_0(s)/Z_1(s) \), then the overall system consisting of the two interacting subsystems is stable.

Notice that the criterion is applied to the system total impedance \( Z_{tot}(s) \) looking into the interconnection port of the
two subsystems (Fig. 1). This should be contrasted with the Middlebrook criterion and its extensions that apply the Nyquist criterion to a minor loop gain defined as 
\[ T(s) = \frac{Z_o(s)}{Z_i(s)} \]
where 
\[ Z_o(s) \]
is the output impedance of the source subsystem and 
\[ Z_i(s) \]
is the input impedance of the load subsystem. The PBSC is more general than the Middlebrook criterion and all its extensions because it overcomes several difficulties encountered with all the previous stability criteria. First of all, the PBSC is directly applicable to multi-converter systems, since it is applied to the parallel combination of all subsystems connected to a bus. Moreover, a change in power flow direction is not a problem, since no assumption of flow direction is made. This is not true for the Middlebrook criterion, where the power flow direction is fixed. Finally, the PBSC lends itself to the design of active damping networks. In particular, the PBSC can be used in conjunction with a recently proposed control strategy for switching converters called PFF control [11-13], to provide an active way to ensure overall system stability and performance without any hardware modification.

III. A SIMPLE EXAMPLE

This section describes how the PBSC can be used to study the improvement of stability of two connected subsystems, consisting of a lightly damped LC input filter \((L = 100\mu H, C = 900\mu F, r = 0.01\Omega)\) and a constant power load \((-R_L = -10\Omega)\), when a third subsystem with impedance \(Z_{\text{damp}}(s)\), a damping network, is added in parallel as shown in Fig. 2. Due to interactions between the two subsystems [13], the undamped system is unstable (Fig. 2 (a)), whereas, when a proper damping impedance \(Z_{\text{damp}}(s)\) is added, the system is stabilized (Fig. 2 (b)). The technique here used differs significantly from all the methods proposed in the past [7-10], since the goal of the introduction of damping impedance \(Z_{\text{damp}}(s)\) is to stabilize the DC bus voltage by modifying the total impedance \(Z_{\text{tot},\text{damp}}(s)\) (parallel combination of all the impedances connected to the bus) only in the frequency range where the original impedance \(Z_{\text{tot,undamp}}(s)\) violates the newly proposed passivity criterion.

Fig. 3 shows how the total impedance looking into the bus is modified by the addition of \(Z_{\text{damp}}(s)\). It is noticeable how the resonant peak responsible for interactions is significantly reduced in magnitude as shown by the arrow in Fig. 3. Moreover, the phase of the damped system \(Z_{\text{tot,damp}}(s)\) has a value of 0° at the resonant frequency and changes smoothly in the frequency range around the resonant frequency where the interaction occurs, whereas the undamped system
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In practice, either for performance or cost reasons, it is usually desirable that the damping impedance have as little effect on the overall impedance as possible. The system in Fig. 2 can be considered a simplified representation of a DC bus distribution system, with $Z_{L\text{-undamp}}(s)$ representing the equivalent input impedance of all feedback-controlled converters connected to the bus. This impedance typically exhibits negative incremental resistance within the feedback control loop bandwidth. Impedance $Z_{damp}(s)$ can be designed in such a way that it modifies the overall system impedance only in the frequency range where the passivity condition was violated. Fig. 5 shows that the following damping impedance

$$Z_{damp}(s) = \frac{1 + \frac{s}{\omega_1}}{\left(\frac{s}{\omega_0} + 1\right)}$$

$$= \left(1 + \frac{s}{2\pi f_{res}}\right)\left(1 + \frac{s}{2\pi \omega_0}\right)$$

$$\left(1 + \frac{s}{2\pi 500}\right)$$

(5)

does not modify the negative input impedance at low frequencies, but it increases the phase of $Z_{L\text{-undamp}}(s)$ up to 0° at the resonant frequency, so that $Z_{tot\text{-undamp}}(s)$ becomes passive guaranteeing bus stability. Notice that impedance $Z_{damp}(s)$ is not passive by itself, since it has a phase of $-180°$ at low frequency.

Finally, simulation results in the time domain are given for both cases (a) and (b) in Fig. 2. A unity step load current change at time $t=0.1s$ is simulated. The undamped case and the damped case are compared in Fig. 6. For the undamped case, impedance $Z_{tot\text{-undamp}}(s)$ violates the passivity condition at the input filter resonance frequency and this causes sustained bus voltage oscillation, as shown by the dashed line in Fig. 6. Let us examine the damped case now. Due to the improvement in the phase of $Z_{tot\text{-damp}}(s)$ around the input filter resonant frequency caused by the addition of damping impedance $Z_{damp}(s)$, the bus voltage is stabilized as shown by the solid line in Fig. 6.

An important conclusion from this simple example is that the PBSC overcomes difficulties of component grouping encountered in previous works based on the Middlebrook criterion and its extensions [10], since the overall system stability is studied by looking at the passivity of the combination of all impedances connected to the bus.

IV. DC POWER DISTRIBUTION SYSTEM EXAMPLE WITH EXPERIMENTAL VALIDATION

This section discusses the compensation method used to improve bus voltage stability of a DC power distribution system.

A. System Description

As shown in Fig. 7, the DC power distribution system consists of two subsystems connected to a common DC bus, i.e., a low-pass LC input filter ($L_f = 2mH$, $C_f = 100\mu F$), and a three-phase DC/AC converter operating at $V_{bus} = 200V$, $V_{abc} = 100V_{pk}$, $f_{SW} = 20kHz$. The input filter can be viewed as representing the lumped impedance of all other converters connected to the bus. A picture of the laboratory setup is shown in Fig. 8. The digital control is implemented using dSPACE DS1104 system, i.e. a DSP based control platform especially designed for rapid control prototyping of high-speed multivariable systems. The DS1104 processor
board contains a 64-bit PowerPC 603e floating-point processor running at 250MHz and a slave-DSP system based on a TMS320F240 DSP microcontroller, providing a complete real-time control package.

### B. PFF Control Design Procedure using PBSC Criterion

As previously stated, the PBSC can be easily coupled with a recently proposed control strategy for switching converters called PFF control [11-13], to provide an active approach to improve stability. The DC/AC converter control has a PFF controller in addition to the conventional negative feedback (NFB) controller as shown in Fig. 7. The complete control schematic diagram of the three-phase DC/AC converter with a lightly damped input filter is reported in Fig. 9. Notice the PFF controller $G_{CFF}$ and the NFB controller $G_{CFB}$. In the following the notation introduced in [13] is used. The effect of the PFF is to modify the DC/AC converter input impedance according to the following equation

\[
\frac{1}{Z_{\text{inFFB}}} = \frac{1}{Z_{\text{inOL}}} \frac{1}{1 + T_{FB}} + \frac{1}{1 + T_{FF}} \frac{1}{1 + T_{FB}} \left( \frac{Z_{\text{inFB}}}{1 + T_{FB}} \right) + \frac{1}{Z_{\text{damp}}}
\]

where

\[ T_{FB} = G_{CFB} \cdot G_{vdd,OL} \]  (7)

and

\[ T_{FF} = G_{CFF} \cdot G_{igd,OL} \]  (8)

are the FB and FF loop gains, respectively. Notice that $T_{FF}$ has dimension of admittance [13]. From Equation (6) it is evident that the PFF control actively introduces the impedance $Z_{\text{damp}}$ in parallel to the already existing $Z_{\text{inFB}}$. Impedance $Z_{\text{damp}}$ is
designed to “passify” the system, i.e., to ensure that passivity of $Z_{tot,FFFB} = Z_{in,FFFB}/Z_{o,IF}$ is satisfied at all frequencies, whilst maintaining good negative input impedance characteristic at low frequencies, so as not to affect closed-loop inverter operation at low frequencies. A damping impedance $Z_{damp}$ that satisfies these requirements is given in Equation (5). Finally, by combining the last part of Equations (6) and Equation (8), the PFF controller is directly calculated as

$$G_{CFF} = \frac{1}{g_{igd,DL} z_{damp}} (9)$$

The PFF control design procedure can be summarized in the following three steps:

1. The FB controller $G_{CFB}$ is designed first to provide the desired output regulation for the case of an ideal source subsystem ($Z_{o,IF} = 0$).

2. The effect of a non-ideal source subsystem ($Z_{o,IF} \neq 0$) on the whole system is analyzed by looking at the Nyquist plot of $Z_{tot,FB} = Z_{in,FB}/Z_{o,IF}$. Either a very large Nyquist contour lying on the RHP or a Nyquist contour lying on the LHP is expected for marginally stable systems or for unstable systems, respectively.

3. The PFF controller $G_{CFB}$ is calculated according to Equation (9) after a good “passifying” damping impedance $Z_{damp}$ has been found. Tuning of $Z_{damp}$, which has the form of Equation (5), is performed by looking at the Nyquist plot of $Z_{tot,FFFB} = Z_{in,FFFB}/Z_{o,IF}$ for bus impedance passivity assessment. Notice that the DC bus voltage stability improvement is directly assessed during the design step, therefore avoiding annoying iterative design procedures.

The first step is to design the FB controller $G_{CFB}$ for the DC/AC converter with no input filter. The FB loop gain $T_{FB}$ is designed to have a control bandwidth of 1kHz and a phase margin of 60°. The second step is to examine the effect of the non-ideal source subsystem. When the input filter in Fig. 9 is added, the Nyquist plot of $Z_{tot,FB} = Z_{in,FB}/Z_{o,IF}$, shown as the dashed green contour in Fig. 10, is a large contour lying in the RHP, indicative of a marginally passive (and marginally stable) system. The large contour radius indicates a significant parallel resonance, which reduces system stability. The third step is to design PFF controller $G_{CFF}$ to ensure overall passivity. Fig. 11 shows how the total bus impedance is modified by combining PFF control with FB control. The original total impedance $Z_{tot,FB} = Z_{in,FB}/Z_{o,IF}$ follows $Z_{o,IF}$ (in the parallel combination the smaller impedance dominates) for all frequencies and exhibits a large resonant peak around 400Hz and a 180° abrupt phase change around the resonance frequency. Notice that for some frequencies the phase of $Z_{tot,FB} = Z_{in,FB}/Z_{o,IF}$ is very close to $+90°$ and $-90°$ index of a marginally passive (and marginally stable) system. By the addition of the PFF control, the total impedance is modified to $Z_{tot,FFFB} = Z_{in,FFFB}/Z_{o,IF}$. The effect is to reduce the resonant peak and to bring the phase at the resonant frequency closer to 0°. In other words, the passivity of the total bus impedance $Z_{tot,FFFB} = Z_{in,FFFB}/Z_{o,IF}$ has improved. The improvement of the bus passivity is also shown in Fig. 10. The approximate diameter of the Nyquist contour is drastically reduced from a diameter of about 1000 to a diameter of about 7.

C. Experimental Results

To verify the stability improvement introduced by the PFF control, experimental results of the FB and the FFFB cases are compared. Figs. 12-13 show the transient responses for the FB case of the DC bus voltage and three-phase output voltage in correspondence of a symmetric three-phase load step from 20Ω to 10Ω and of a voltage reference step from 90Vpk to 45Vpk, respectively. Figs. 14-15 show the same transients for the FFFB case, when the PFF control is added. Let us examine the FB case first in Figs. 12-13. In both cases the bus voltage exhibits lightly damped oscillations due to the fact that the passivity condition is only marginally met at the resonant frequency, where the interaction between the two subsystems occurs. Let us examine the case of the addition of the PFF control now. When the PFF control is added, the desired active damping of the voltage bus is obtained and the bus
impedance is made passive. As a result, the voltage bus is stabilized as Figs. 14-15 show. Notice that the three-phase output voltage is somehow more distorted during the transient. This is due to the fact that the PFF control slightly reduces the FB bandwidth as previously demonstrated in [13]. However, Figs. 14-15 show that a good trade-off between stability improvement and FB control bandwidth reduction has been achieved.

V. CONCLUSIONS

The PBSC has been proposed as a new stability criterion based on the passivity of the overall bus impedance. If that impedance is passive then the system is stable. This criterion overcomes several difficulties encountered in existing stability criteria, such as the artificial conservativeness of the Middlebrook criterion, and the sensitivity to component grouping of the ESAC criterion. Moreover, it is readily applicable to the case of multi-converter systems even when the power flow direction may change due to system reconfiguration.

A simple example and a more complex DC distribution system example have shown the powerfulness of the criterion and how it can be used for the design of damping networks, so that overall system passivity, and therefore stability, is guaranteed. Unlike all existing stability criteria, the PBSC is directly oriented towards the design of damping impedances that “passify” and therefore stabilize the bus voltage. In particular, in the DC distribution system example, it was shown that the PBSC can be coupled with the PFF control, a recently proposed control strategy for switching converters, to provide an active approach to improve stability. A design procedure that ensures passivity and therefore stability has been developed and presented. The PBSC coupled with the PFF control results in a great improvement in bus voltage stability.
APPENDIX

Parseval’s Theorem:

\[
\int_{-\infty}^{\infty} v(t)i(t)dt =
\frac{1}{2\pi} \int_{-\infty}^{\infty} V(j\omega) e^{j\omega t} d\omega i(t)dt =
\frac{1}{2\pi} \int_{-\infty}^{\infty} V(j\omega) I(-j\omega)d\omega =
\frac{1}{2\pi} \int_{-\infty}^{\infty} V(j\omega) I^*(j\omega)d\omega
\]

ACKNOWLEDGMENT

This work was supported by the Office of Naval Research under grant N00014-08-1-0080.

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