Positive Feed-Forward Control of Three-Phase Voltage Source Inverter for DC Input Bus Stabilization

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Abstract—A Positive Feed-Forward (PFF) controller is proposed as a new active approach to improve stability of a three-phase DC/AC switching inverter. In particular, the proposed approach solves the source subsystem interaction problem, i.e., a system stability degradation which is commonly observed when the inverter is connected to a DC voltage source subsystem that presents finite Thévenin impedance. The implemented strategy is to combine the PFF control with the conventional Negative Feedback (NFB) control. When properly designed, the PFF control modifies the inverter input impedance in the frequency range where the subsystem interaction occurs. As a result, the PFF control stabilizes the DC bus voltage ensuring overall system stability, while allowing the NFB control to maintain good output voltage regulation performance. The approach results in greatly improved stability and damping of the DC bus voltage with a slight reduction of output feedback control bandwidth.

I. INTRODUCTION

DC distribution systems have been investigated as an alternative to conventional AC distribution systems for industrial applications [1-2] and for military applications such as the power distribution system for the all-electric ship proposed by the US Navy. Advances in power electronics technology and the impact of renewable distributed generation sources lead to the consideration of a DC distribution system for high reliability, interface flexibility, and high power density. However, one of the challenges for a DC distribution system is potential stability degradation due to the presence of Constant Power Loads (CPLs). Typically, feedback-controlled converters and inverters behave as CPLs at their input terminals within the bandwidth of the system control loop gain [3-5]. The CPLs create the so-called negative incremental input impedance, which is the origin of the undesired destabilizing effect and cause of the subsystem interaction problem [6]. Although each subsystem is independently designed to be standalone stable, a system consisting of many power-electronics-based subsystems may exhibit degraded stability due to subsystem interactions caused by CPLs.

In this paper, the focus is on the source subsystem interaction problem, i.e., a degradation of the closed-loop stability of a switching converter when it is fed by a DC voltage source subsystem with finite Thévenin impedance. The PFF control technique is proposed here as an active damping approach to improve stability which is degraded due to this type of interaction. This approach to mitigate the source subsystem interaction is innovative with respect to previous approaches, which mostly depend on passive solutions, such as a damping circuit or a large dc-link capacitor. The PFF control technique has so far been applied only to DC/DC converters under Voltage Mode and Peak-Current-Mode control [7-8]. However, only a simplified frequency-domain PFF controller design procedure based on constraints of feedback (FB) and feed-forward (FF) control loop gains at low and high frequencies has been presented. In the present work, the PFF control technique is extended to DC/AC inverters and an improved PFF controller design procedure is proposed for the mid-frequency region, which is the actual region where the source subsystem interaction occurs.

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A lightly damped input filter is connected to a three-legged Voltage Source Inverter (VSI) with LC low-pass filters and a balanced three-phase resistive load to demonstrate how the poor stability of the system under FB control only can be greatly improved by adding the PFF control. A complete small-signal model of the VSI in open-loop, FB, FF, and both Feed-Forward and Feedback (FFFB) closed-loop systems is derived on the dq rotating reference frame using g-parameter representation [9]. The design procedure for the PFF controller is discussed in detail with particular focus on the mid-frequency region. The source subsystem interaction problem and stability improvement by passivity concepts at the inverter input terminals. Simulation results validate the PFF control approach.

II. SMALL-SIGNAL MODELING FOR THREE-PHASE VSI

A complete small-signal model for the three-phase VSI in Fig. 1 is presented in this section based on dq rotating reference frame using g-parameter representation [9]. The first step is to model the stand-alone inverter, i.e., the inverter supplied by an ideal DC voltage source, by applying the dq transformation to all averaged state variables. Then, small-signal open-loop inverter model is described. Finally, closed-loop models for FB, FF and FFFB control are derived.

As in [11], an equivalent averaged model in dq rotating coordinates can be obtained from the averaged abc-model of a VSI with LC low-pass filters and balanced three-phase resistive load by using the Park Transformation. This procedure yields the following large-signal averaged equations:

\[ C \frac{dv_q}{dt} = i_q - \frac{v_q}{R} + \alpha Cv_q \]  \hspace{1cm} (1)
\[ L \frac{di_d}{dt} = d_q v_g - v_d + \alpha Li_d \]  \hspace{1cm} (2)
\[ C \frac{dv_d}{dt} = i_d - \frac{v_d}{R} - \alpha Cv_d \]  \hspace{1cm} (3)
\[ L \frac{di_q}{dt} = d_q v_g - v_q - \alpha Li_q \]  \hspace{1cm} (4)

where \( v_d, v_q, i_d, i_q \) are the state variables and \( d_d, d_q \) are the control input variables.

The model so obtained contains cross-coupling terms between the \( d \) and \( q \) channels [9-10], as shown by the shaded blocks in Fig. 2. Consequently, the resulting system is a fourth-order MIMO system. A decoupling technique, which is an improvement of the technique proposed in [10], is implemented to get two second-order decoupled SISO systems. The technique consists of a decoupling block placed in front of the averaged model depicted in Fig. 2, which implements the following equations:

\[ d_d = d_d' - \frac{\alpha k}{v_g} \left( 2i_q - \frac{V}{R} - \alpha Cv_d \right) \]  \hspace{1cm} (5)
\[ d_q = d_q' - \frac{\alpha k}{v_g} \left( 2i_d - \frac{V}{R} + \alpha Cv_q \right) \]  \hspace{1cm} (6)

Then, after decoupling, perturbation and linearization can be applied to obtain the small-signal open-loop (OL subscript) VSI model (7). Due to decoupling, the \( d \)-axis equation is a function of \( d \)-axis quantities only and the \( q \)-axis equation of \( q \)-axis quantities only.

\[
\begin{bmatrix}
\dot{i}_d \\
\dot{v}_d \\
\dot{i}_q \\
\dot{v}_q \\
\end{bmatrix} =
\begin{bmatrix}
1 & G_{gdd,OL} & G_{gqd,OL} & 0 \\
0 & 0 & G_{gdd,OL} & G_{gqd,OL} \\
0 & 0 & 0 & -Z_{out,eq,OL} \\
\end{bmatrix}
\begin{bmatrix}
\dot{v}_d \\
\dot{v}_q \\
\dot{i}_d \\
\dot{i}_q \\
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
-\frac{V}{Z_{in,OL}} \\
\end{bmatrix}
\]  \hspace{1cm} (7)

The open-loop transfer functions in model (7) are given by Equations (8)-(14). Notice also that, due to the decoupling, the whole system is equivalent to two independent DC/DC buck converters.

\[
\frac{1}{Z_{in,OL}} = \frac{3}{2} \frac{D_a^2}{R} \left( \frac{1 + sRC}{1 + s \frac{L}{R} + s^2LC} \right)
\]  \hspace{1cm} (8)

\[
G_{gdd,OL} = \frac{3V_g}{2R} \left( \frac{I_d R}{V_g} + \frac{D_a (1 + sRC)}{1 + s \frac{L}{R} + s^2LC} \right)
\]  \hspace{1cm} (9)
where \( D_d = V_o/V_g \), \( D_q = 0 \), \( I_d = V_o/R \), and \( f_q = \omega C V_o \) are the steady-state duty cycle and inductor currents.

Finally, the closed-loop VSI model (16) for the combined feed-forward and feedback (FFFFB subscipt) control of Fig. 1 is derived by substitution of (15) into the open-loop model (7).

\[
\begin{bmatrix}
\dot{d}_d \\
\dot{d}_q
\end{bmatrix} = -G_{CFB} \begin{bmatrix}
\ddot{v}_d \\
\ddot{v}_q
\end{bmatrix} + G_{CFF} \begin{bmatrix}
\dot{v}_d \\
\dot{v}_q
\end{bmatrix} + \begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

where \( G_{CFB} \) and \( G_{CFF} \) are the transfer functions of the FB controller and PFF controller, respectively.

The transfer functions in (16) are given by (17)-(21).

\[
G_{g_{dpq,OL}} = \frac{3}{2} \frac{V_g}{R} \left( \frac{sR}{V_g} \right) \quad (10)
\]

\[
G_{g_{id,OL}} = \frac{3}{2} D_d \frac{1}{1 + sL + s^2LC} \quad (11)
\]

\[
G_{g_{sdq,OL}} = \frac{D_d}{1 + sL + s^2LC} \quad (12)
\]

\[
G_{g_{dd,OL}} = G_{g_{dpq,OL}} = \frac{V_g}{1 + sL + s^2LC} \quad (13)
\]

\[
Z_{out,dd,OL} = Z_{out,qq,OL} = \frac{sL}{1 + sL + s^2LC} \quad (14)
\]

where \( D_d = V_o/V_g \), \( D_q = 0 \), \( I_d = V_o/R \), and \( f_q = \omega C V_o \) are the steady-state duty cycle and inductor currents.

From the expression for the input impedance (17), the effect of PFF control is evident: it provides a way to control the VSI input impedance through the last term on the right hand side of (17). In particular, impedance with positive real part is desired to provide damping in the frequency range where source subsystem interaction occurs. Therefore, the controller behaves like an active filter providing damping for the input filter at the input of the VSI.

### III. The Positive Feed-Forward Control

#### A. The Concept of PFF Control

The source subsystem interaction problem manifests itself as DC input bus instability when a lightly damped source subsystem is connected to the VSI operating under FB control, which exhibits negative incremental input impedance \( Z_{in,FB} \). The goal of the proposed PFF control, as shown in Fig. 3b, is to stabilize the DC input voltage by modifying, in the frequency range where the source interaction occurs, the VSI input impedance so that it exhibits a positive real part. This approach is conceptually different from conventional Negative Feed-Forward control [12], which is commonly introduced to compensate for input voltage variations, so that the output voltage is not affected.
As a result, Negative Feed-Forward control actually has a destabilizing effect at the input port of a converter by extending negative input impedance up to higher frequencies.

Since FB control loop gain should be large within the control bandwidth, in [6-7] the PFF controller was designed to be a high-pass filter (HPF), so that the PFF control loop gain dominates at high frequencies to reduce the effect of source subsystem interaction, while leaving the FB control loop gain dominating at low frequencies to provide tight output voltage control. Fig. 4 shows how expression (17) for the input impedance under FFFB control simplifies at low frequencies and at high frequencies according to these constraints. At low frequencies, where FB control loop gain dominates (|\(T_{FF}\)|<|\(T_{FB}\)|), \(Z_{in\_FFFB}\) can be approximated by the input impedance under FB control \(Z_{in\_FB}\), which exhibits negative real part at low frequency. This is the origin of potential instability as explained above. On the other hand, at high frequencies, where FF control loop gain dominates (|\(T_{FF}\)|>>|\(T_{FB}\)|), the value of \(Z_{in\_FFFB}\) is determined by the positive feed-forward term \(T_{FF}\), which can mitigate the negative input impedance under FB control.

However, the above description is somewhat simplified. In fact, the stabilization effect tends to occur in the mid-frequency region and is attributable to an improvement in phase margin, which is not captured by the FF and FB loop gain magnitude conditions in Fig. 4. Therefore, in addition to the constraints explained above, the new contribution here for the PFF controller \(G_{CFF}\) design is to provide design criteria valid in the mid-frequency region, where the source subsystem interaction occurs and where FF and FB control loop gains are comparable. In particular, by using a passivity-oriented approach [13], the controller \(G_{CFF}\) can be designed to provide both

- the desired active damping for the input filter, and
- the desired stability phase improvement for the VSI input impedance.

The following subsection shows how these two goals can be accomplished by placing two poles \(f_{p1}\) and \(f_{p2}\) of controller \(G_{CFF}\) in the mid-frequency region based on DC input bus port passivity considerations.

B. Passivity-Oriented PFF Control Design Procedure

Recalling that a linear time invariant 1-port is passive if the impedance \(Z(s)\) looking into the port has:

- no right half plane (RHP) poles;
- a Nyquist contour which lies totally within the closed RHP.

The previous definition has an important consequence: the phase of \(Z(j\omega)\) must be between -90° and +90° at all frequencies. The phase of \(Z(j\omega)\) is the difference between the phase of the voltage \(v(j\omega)\) applied to the port and the phase of the current \(i(j\omega)\) injected into the port. If the phase of \(Z(j\omega)\) is between -90° and +90°, the average power into the port is positive and therefore the system consumes energy (it is a passive system). If the phase is +90° or -90°, the average power is zero, and the system is lossless. If the phase is less than -90° or greater than +90°, the average power is negative and the system produces energy (it is an active system).

The PFF design procedure proposed here is based on imposing the above stated passivity condition to the DC input bus port of Fig. 3. Specifically, the controller \(G_{CFF}\) is designed by looking at the Nyquist plot of the total port impedance \(Z_{tot}=Z_{out\_ZS}/Z_{in\_CL}\), i.e., the output impedance of the input filter in parallel with the input impedance of the VSI under closed-loop operation (CL subscript could be either FB or FFFB). In this way one can assess the passivity and also stability of the DC input bus port during the choice of the controller design parameters. It is desired that the Nyquist plot of \(Z_{tot}\) lies in the RHP as close as possible to the origin so that the system has a good passivity margin (it is far away from the passivity boundary) and it is well damped. However, the desired improvement in the impedance \(Z_{tot}\) is needed only in the frequency range where the source subsystem interaction occurs, so that at lower frequency good output regulation is maintained. By doing so, the DC input bus is actively well damped and the PFF control solves the source subsystem interaction problem. Also a good trade-off between stability improvement and dynamic output performance is obtained.

The proposed design procedure consists of the following steps:

1. The FB controller \(G_{CFB}\) is designed to provide the desired output regulation for the case of an ideal source subsystem (\(Z_{out\_ZS}=0\)).
2. The effect of a non-ideal source subsystem (\(Z_{out\_ZS} \neq 0\)) on the whole system is analyzed by looking at the Nyquist plot of \(Z_{tot}=Z_{out\_ZS}/Z_{in\_FB}\). Either a very large Nyquist contour lying on the RHP or a Nyquist contour lying on the left half plane (LHP) is expected for marginally stable systems or for unstable systems, respectively.
3. The PFF controller \(G_{CFF}\) is designed as a simple HPF so that \(T_{FF}\) satisfies the inequalities reported in Fig. 4. By looking at the Nyquist plot of \(Z_{tot}=Z_{out\_ZS}/Z_{in\_FFFB}\), the HPF gain \(G_{\omega}\) and corner frequency \(f_{p1}\) should be chosen to improve passivity condition for the DC input bus port.
4. A second pole \( f_{p2} \) placed close to \( f_{p1} \) helps to obtain the best passivity condition. The reason for the addition of \( f_{p2} \) will be explained in the next section.

5. DC input bus stability can also be assessed by applying the Nyquist criterion to the minor loop gain \( T_{MLG} = \frac{Z_{out} Z_{in}}{Z_{in}} \) according to the unified impedance criterion for the case of source subsystem interaction only as defined in [7-8].

IV. SIMULATION RESULTS

A three-phase VSI with an input filter as source subsystem shown in Fig. 5 is used to illustrate the effectiveness of the PFF control in DC bus voltage stabilization. The parameters of the system are also reported in Fig. 5.

The first step is to design the FB controller \( G_{CFB} \) for the VSI with no input filter. The FB loop gain \( T_{FB} \) is designed to have a control bandwidth of 1kHz and a phase margin of 60°. However, when the input filter in Fig. 5 is added, the Nyquist plot of \( Z_{tot} = Z_{out} Z_{in} \) shown as the dashed green contour in Fig. 6, is a large contour lying in the RHP, index of a marginally passive (and marginally stable) system. To improve the passivity, degraded by the source subsystem interaction, PFF controller \( G_{PFF} \) is designed as a simple HPF following the loop gain constraints in Fig. 4. The gain \( G_{\infty} \) and corner frequency \( f_{p1} \) of the HPF are gradually increased from very low values until the Nyquist contour of \( Z_{tot} \) is reduced drastically, as also shown by the solid red contour in Fig. 6. The approximate diameter of the Nyquist contour is reduced from a value of 28 down to a value of 2.8. This contour represents the optimal (smallest) contour obtainable with the simple HPF. Further increasing \( G_{\infty} \) and \( f_{p1} \) actually causes the Nyquist contour to start increasing again. However, the stability improvement obtained with the simple HPF is poor. Fig. 7 shows how \( Z_{tot} \) is modified by combining PFF control with FB control. The original total impedance \( Z_{tot} = Z_{out} Z_{in} \) follows \( Z_{out} \) (in the parallel combination the smallest impedance dominates) for all frequencies and exhibits a large resonant peak around 500Hz. The total impedance is modified in the mid-frequency region by the introduction of the HPF (examine \( Z_{tot} = Z_{out} Z_{in} \) in Fig. 7). However, the resonance peak responsible for interactions is simply shifted to lower frequency and slightly reduced in magnitude, and the phase of the VSI input impedance \( Z_{in} \) at resonance has increased to approximately -90°. It is desirable for better stability to further reduce the resonant peak and to bring the phase at the resonant frequency closer to 0°.

Therefore, a second pole \( f_{p2} \) is added at a frequency close to \( f_{p1} \) to further improve DC input bus passivity, as shown in Fig. 8. The approximate diameter of the Nyquist contour is further reduced from a diameter of 2.8 to a diameter of 0.5. The source subsystem interaction problem is solved by actively damping the resonant peak as shown in Fig. 9. Moreover, the phase of VSI input impedance has improved and is now close to 0°, which means that the VSI now exhibits an input impedance with positive real part in the frequency range where the source subsystem interaction...
occurs. (Refer to the lumped system representation reported in Fig 3.)

The Bode plot of the designed PFF controller is shown in Fig. 10. The figure also shows the three regions: the FB region where \( T_{FB} \) dominates, the mid-frequency region where \( T_{FB} \) and \( T_{FF} \) are comparable and where the source subsystem interaction occurs, and the FF region where \( T_{FF} \) dominates. Notice that the maximum frequency of interest for control is the Nyquist frequency, which is \( 6kHz \) equal to one half of the switching frequency. (Refer to Fig. 4.)

Finally, simulation results in the time domain are given. A step load change from \( 8\Omega \) to \( 4\Omega \) at time \( t=0.1s \) is simulated. The FB case and the FFFB case are compared in Fig. 11 and Fig. 12. For FB control only, the interaction between the input filter and the inverter causes sustained DC input voltage oscillation due to negative inverter input impedance at the input filter resonant frequency as shown by the dashed line in Fig. 11. In Fig. 12, the three-phase output voltage shows a small transient at time \( t=0.1s \) due to the low audio susceptibility of the FB control. Let us examine the PFF case now. Due to the active damping around the resonant frequency of the input filter introduced by the PFF control, the DC input voltage of the inverter is highly stabilized as shown by the solid line in Fig. 11. The corresponding output voltage of Fig. 12 is somewhat more distorted during the transient because the controller behavior is modified by the PFF controller in the mid-frequency
region. This shows a good trade-off between stability and dynamic performance.

The stability improvement can be analyzed by applying the Nyquist criterion to the minor loop gain $T_{MLG}=Z_{out\_ZS}/Z_{in\_CL}$ which gives information on stability of the interconnected system [7-8]. Nyquist plots of the minor loop gain for the FB case only and the FFFB case are compared in Fig 13. While the FB case has infinite phase margin but a small gain margin, the FFFB case has both a very large gain margin and a good phase margin equal to $57.6^\circ$. Notice that the PFF acts so that the non-interaction condition $|Z_{out\_ZS}|<<|Z_{in\_FB}|$ [14] is violated, but it has the benefit of improving the phase margin. Another way to analyze the stability improvement is to look at the sensitivity function [15], defined as

$$S_{f\_CL} = \frac{1}{1 + T_{MLG}}$$  \hspace{1cm} (22)

where CL could be either FB case or FFFB case. The inverse of $S_{f\_CL}$ is the distance from a point of the Nyquist contour of $T_{MLG}$ to the critical point $-1$. Fig. 14 shows the comparison between $S_{f\_FB}$ and $S_{f\_FFFB}$. The maximum of $S_{f\_FB}$ occurs at the resonant frequency of the input filter in the mid-frequency region, index of a small distance from the point $-1$. For the PFF case, the sensitivity function $S_{f\_FFFB}$ has been significantly reduced for that frequency revealing a good robustness against instability due to source subsystem interaction.
Finally, Fig. 15 shows how the FB loop gain $T_{FB}$ is modified by the introduction of the input filter and the PFF control. The introduction of the input filter lowers the bandwidth from $1\text{kHz}$ to $523\text{Hz}$ and the phase margin from $60^\circ$ to $25^\circ$. The introduction of the PFF control slightly reduces the FB bandwidth to $370\text{Hz}$. Notice that the PFF control increases the glitch in the magnitude plot, index of a even stronger interaction, but has the benefit of increasing the phase margin to $51^\circ$. This shows that the PFF control has been designed to show a good compromise between stability improvement and FB bandwidth reduction.

V. CONCLUSION

The PFF control is proposed as a new active approach to improve stability of a three-phase VSI. The proposed approach solves the source subsystem interaction problem, i.e., a stability degradation which is observed when the inverter is connected to a DC voltage source subsystem which presents finite Thévenin impedance.

The PFF controller has been designed by looking at the passivity condition of the DC input port. This passivity-oriented design procedure is straightforward and one can gain insight into how the inverter input impedance should be modified in order to ensure overall system stability. As a result, the inverter input impedance has been modified in the mid-frequency region where the source subsystem interaction occurs, while allowing the FB controller to tightly regulate the output voltage.

Small-signal models for the OL, FB, and FFFB VSI have been presented in $dq$ rotating frame using $g$-parameter representation. The system stability has been analyzed based on the unified impedance criterion for the case of source subsystem interaction only.

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